## Simple Harmonic Motion

Definitions

Simple harmonic motion: Any motion that repeats itself after a certain period is known as a **periodic** motion, and since such a motion can be represented in terms of sines and cosines it is called a **harmonic** motion.

Simple harmonic motion (S.H.M. for short) is the name given to a particular type of harmonic vibration. The following are examples of SHM:
- a test-tube bobbing up and down in water
- a simple pendulum
- a compound pendulum
- a vibrating spring
- a vibrating cantilever
- a marble on a concave surface
- liquid oscillating in a U-tube

SHM requires the same magnitude of force irrespective of direction of displacement but is a restoring force dependent on the amount of displacement, y, from the equilibrium position:
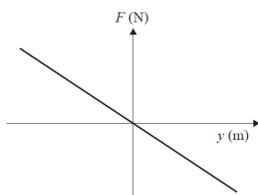
Equations

$$a = -\omega^2 y$$

	Acceleration	a	m s^{-2}
	Angular velocity	ω	rad s^{-1}
	Displacement from equilibrium	y	m

$$F = -ky$$

	Restoring force	F	N
	constant	k	N m^{-1}
	Displacement from equilibrium	y	m

$$y = A\sin\omega t$$
$$y = A\cos\omega t$$

	Displacement from equilibrium	y	m
	Angular velocity	ω	rad s^{-1}
	time	t	s

$$v = A\omega\cos\omega t$$
$$v = -A\omega\sin\omega t$$

	velocity	v	m s^{-1}
	Amplitude	A	m
	Angular velocity	ω	rad s^{-1}
	time	t	s

$$a = -A\omega^2\sin\omega t$$
$$a = -A\omega^2\cos\omega t$$

	Acceleration	a	m s^{-2}
	Amplitude	A	m
	Angular velocity	ω	rad s^{-1}
	time	t	s

Questions
GRAVITY ELEVATORS (2015;2)

Earth's average radius = 6.38×10^6 m.

In the 2012 science fiction movie *Total Recall,* a gravity- powered elevator called "The Fall" is used to transport passengers between the Northern and Southern hemispheres, straight through the Earth. If a straight tunnel could be dug through the Earth from North Pole to South Pole, an elevator could be used to harness the gravity of the planet. Once dropped, the elevator would accelerate downwards and then decelerate once it had passed through the midpoint and -in the absence of friction - would just arrive at the far side of the Earth. An equation can be used to summarise the acceleration of the elevator.

$$a = -1.54 \times 10^{-6}\ y$$

where y = distance from the midpoint

(b)   Calculate:

(i)   The maximum acceleration of the elevator.

(ii)   The maximum linear velocity of the elevator.

(c)   Explain how the equation given shows that the elevator is undergoing simple harmonic motion.

(d)   Calculate the time the journey from the North Pole to the South Pole would take.

Terms

Tips

All of the equations involving sin ωt and cosωt can be simplified when calculating maximum displacement, velocity and acceleration.

Sin ωt = 1, cos ωt = 1 when calculating maximum values.

$$T = \frac{2\pi}{\omega}$$

	Time period	T	s
	Angular velocity	ω	rad s^{-1}

Is again useful

Answers

(b) $a = -1.54 \times 10^{-6}\,y$, $y = A$ and $\omega^2 = 1.54 \times 10^{-6}$
$a_{max} = -A\,\omega^2 = 1.54 \times 10^{-6} \times 6.38 \times 10^6 = 9.83$ m s^{-2}

(c) $v_{max} = A \times \omega = 6.38 \times 10^6 \times \sqrt{1.54 \times 10^{-6}}$
$v_{max} = $ **7910 m s^{-1}**

(d) $f = \frac{1}{T}$ and $\omega = 2\pi\,f$ and $\omega^2 = 1.54 \times 10^{-6}$
so
$$T = \frac{2\pi}{\omega}$$
$$T = \frac{2\pi}{\sqrt{1.54 \times 10^{-6}}}$$
$T = 5063$ s $= 84$ minutes

Journey will take ½T so the time = 42 minutes