Angular Motion

Definitions

Rotation – or Angular motion – can best be described in terms of angular displacement, time, angular velocity, and angular acceleration.

The angular displacement of one revolution can mathematically be represented by 2π radians meaning that angular velocity can be converted from revolutions per minute into radians per second by

$$1rpm = \frac{2\pi}{60} rads^{-1}$$

Torque or turning force causes angular acceleration.

If we want to make a wheel rotate, we exert a tangential force on its rim. The turning effect of a force or torque or moment of a force:

Equations

$\omega = \frac{\Delta\theta}{\Delta t}$	Angular velocity	ω	rad s^{-1}
	Change in angular displacement	Δθ	rad
	Change in time	Δt	s
$\alpha = \frac{\Delta\omega}{\Delta t}$	Angular acceleration	α	rad s^{-2}
	Change in angular velocity	Δω	rad s^{-1}
	Change in time	Δt	s
$\omega = 2\pi f$	Angular velocity	ω	rad s^{-1}
	Frequency	f	Hz
$\omega_f = \omega_i + \alpha t$	Final angular velocity	ω_f	rad s^{-1}
	Initial angular velocity	ω_i	rad s^{-1}
	Angular acceleration	α	rad s^{-2}
	Time	t	s
$\theta = \frac{\omega_f + \omega_i}{2} t$	Angular displacement	θ	rad
	Final angular velocity	ω_f	rad s^{-1}
	Initial angular velocity	ω_i	rad s^{-1}
	Time	t	s
$\omega_f^2 = \omega_i^2 + 2\alpha\theta$	Final angular velocity	ω_f	rad s^{-1}
	Initial angular velocity	ω_i	rad s^{-1}
	Angular acceleration	α	rad s^{-2}
	Angular displacement	θ	rad
$\theta = \omega_i t + \frac{1}{2}\alpha t^2$	Angular displacement	θ	rad
	Initial angular velocity	ω_i	rad s^{-1}
	Time	t	s
	Angular acceleration	α	rad s^{-2}
$\tau = Fr$	Torque	τ	N m
	Force	F	N
	Radius of arc/circle	r	m

Motion problems can be solved using the 4 equations of motion:
You will be given 3 variables and asked to work out a fourth variable.
Choose the equation that only has these four (or the equation without the fifth variable).

Questions

ROTATIONAL MOTION AT THE PLAYGROUND (2022;1)

Some children are playing on a roundabout. Riley pushes the roundabout and gets it spinning at a constant angular speed. It makes one revolution in 1.23 s.

(a) Show that the angular velocity of the roundabout is 5.11 rad s^{-1}.

(b) Riley stops pushing, and 30.0 s later the roundabout has slowed, so that it takes 2.04 s to make one revolution. The roundabout can be approximated to a spinning disc with a rotational inertia of 57.6 kg m^2.

- Determine the average angular deceleration of the roundabout as it slows.
- Calculate the average frictional torque acting on the roundabout as it slows.

Terms

Tips

- Multiply angular quantity by radius to convert to tangential linear quantity

$d = r\theta$	displacement	d	m
	Radius of arc/circle	r	m
	Angular displacement	θ	rad
$v = r\omega$	velocity	v	m s^{-1}
	Radius of arc/circle	r	m
	Angular velocity	ω	rad s^{-1}
$a = r\alpha$	acceleration	a	m s^{-2}
	Radius of arc/circle	r	m
	Angular acceleration	α	rad s^{-2}

Answers

(a) $\omega_i = \frac{2\pi}{T} = \frac{2\pi}{1.23} = 5.11 \text{ rad s}^{-1}$

(b) $\omega_f = \frac{2\pi}{T} = \frac{2\pi}{2.04} = 3.08 \text{ rad s}^{-1}$

$\alpha = \frac{\Delta\omega}{\Delta T} = \frac{3.08 - 5.11}{30.0} = -0.0677 \text{ rad s}^{-2}$

$\tau = I\alpha = 57.6 \times -0.677 = -3.898 \text{ N m}$