Centripetal Motion

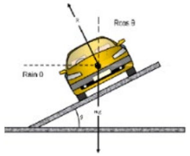
Definitions

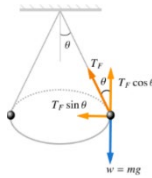
Banked Tracks

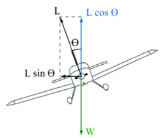
A car on a banked curve must experience a centripetal force if it is moving in an arc (part of a circle).

The Reaction force can be divided into vertical and horizontal components. R sin θ provides the centripetal force to make the car move in a horizontal arc/circle. R cos θ must be equivalent to the vertical weight force.

This same Physics theory can be applied to conical pendulums and aircraft banking during a turn.







Equations

Equation	Quantity	Symbol	Unit
$F_c = \frac{mv^2}{r}$	centripetal force	F_c	N
	mass	m	kg
	velocity	v	m s^{-1}
	radius	r	m
$F = ma$	Force	F	N
	mass	m	kg
	acceleration	a	m s^{-2}

Questions

QUESTION ONE (2020;1)

Tom flies a Boeing 737 airplane with a take-off mass of 7.50 x 10^4 kg. There are times when he flies it horizontally in a straight line, and there are times when he has to take a circular path such that the plane is banked at an angle to the horizontal. The diagrams below represent these two situations.

(a) Draw the force due to gravity and the lift force on the plane in the two situations below.

(b) Compare the size of the force due to gravity and lift force on the plane when Tom flies it horizontally in a straight line, and when he flies it in a horizontal circle banked at an angle. Give reasons why they are similar or different in each situation. *Numerical working is not necessary.*

(c) On one occasion Tom flies the plane of mass 7.50 x 10^4 kg in a circular path, with a speed of 54.0 m s^{-1}, banked at an angle of 35.0° to the horizontal. Calculate the radius of the circle that the plane describes.

Terms

Centripetal force: Force required to keep an object moving in a circle.

Circular motion: Motion in a circle caused by a resultant force acting towards the centre of the circle.

Vector diagram: Scale diagram to show magnitude and direction of vectors

Tips

- The lift force acts 90° to the surface/ 90° to the wings.
- For banked tracks, turning aircraft and conical pendula, the solution involves equating the centripetal force ($F_c = mv^2/r$) and the weight force (F = mg) with some basic trigonometry. E.g. for the banked track:

$$R\sin\theta = \frac{mv^2}{r} \qquad R\cos\theta = mg$$

$$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2/r}{mg}$$

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

This is the exact velocity required for the car to go in a circle. If the car moves faster than this speed, it will slip up the banked curve, and if it goes slower, it will slip down the bank (assuming the road is perfectly frictionless).

- With a conical pendulum going at a constant speed, at a fixed radius, why is the tension in the string greater than the weight force?
The string exerts a force on the pendulum to cause it to change direction. This force shows up as the horizontal component of the tension force. This component combined with the vertical component of tension which is equal to the weight force creates a vector which is bigger than the weight force.

Answers

(a)

(b) In the horizontal position, gravity force = reaction (support) force, andforces are balanced. When flying in a circle, the gravity force remains the same, but the lift force increases. This is because the horizontal component of the lift forceprovides the centripetal force for circular motion. Vertical component of lift = gravity force. Horizontal component = F_c so overall lift force increases when adding the two vector quantities.

(c)

$$F_c = \frac{mv^2}{r} = F_{lift}\sin\theta$$

$$F_g = mg = F_{lift}\cos\theta$$

$$\frac{F_c}{F_g} = \frac{v^2}{rg} = \tan\theta$$

$$r = \frac{v^2}{g\tan\theta} = \frac{54.0^2}{6.869} = 425\text{ m}$$