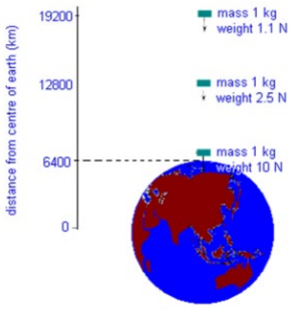
Circular Motion and Gravitational Forces

Definitions

At the earth's surface, the gravitational field strength (*g*) is about 9.81 N kg^{-1} so an object with a mass of 60 kg, such as a person, has a weight of 60 x 9.81 = 587 N.

Weight is the force exerted on a mass by other masses.



Newton's Universal Law of Gravitation: Every mass attracts every other mass.
We don't notice it for most masses because this force is tiny in size, due to the value of G, the universal constant of gravitation:
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Equations

Using just these equations:

$F_g = \frac{GMm}{r^2}$	Gravitational Force	F_g	N
	Universal Gravitational constant	G	N m^2 kg^{-2}
	mass	M, m	kg
	Distance between centre of masses	r	m
$F_c = \frac{mv^2}{r}$	centripetal force	F_c	N
	mass	m	kg
	velocity	v	m s^{-1}
	radius	r	m

Physicists can derive:

- Orbital velocity
- Escape velocity
- Geostationary orbits
- Kepler's laws

Questions

QUESTION ONE (2018;1)

Mass of the Earth = 5.97×10^{24} kg
Radius of the Earth = 6.37×10^{6} m

The Electron Rocket developed by New Zealand company Rocket Lab has begun commercial launches of satellites from the Mahia Peninsula in Hawke's Bay. The rocket can carry a satellite of mass 1.50×10^{2} kg to a stable, circular orbit 5.00×10^{5} m above the Earth's surface.

(a) Show that the force due to gravity on the satellite in this orbit is 1270 N.

(b) The rocket, the satellite, and any space debris at the same altitude in stable, circular orbits, will all travel at the same speed. Show that this is always true by deriving the formula for orbital velocity shown $v_{orbit} = \sqrt{\frac{GM}{r}}$ and use this to determine the orbital velocity of the satellite.

Terms

Mass: The 'quantity of matter' in an object and is a measure of how hard it is to accelerate the object.
Weight: A force is produced on all masses near the earth's surface by our planet's gravitational attraction acting towards the centre of mass of the Earth.
Newton's Universal Law of Gravitation: The Force between two particles is proportional to their masses & inversely proportional to the square of the distance between them.

Tips

- You are not provided with this equation, but it might prove very useful (particularly with derivations):

$v = \frac{2\pi r}{T}$	velocity	v	m s^{-1}
	radius	r	m
	Time for one full rotation	T	s

- **What determines the speed of an object in orbital motion?**
Orbital motion requires that the gravitational force acting on the object be completely utilised in supplying the centripetal force needed at that radius. Equating centripetal force with gravitational force will result in a specific speed for a specific radius.

Answers

(a)

$$F = \frac{GMm}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 150.0}{(6.37 \times 10^{6} + 5.00 \times 10^{5})^2} = 1265.5 \text{ N}$$

$$= 1270 \text{ N (3 sf)}$$

(b)

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}} = 7.613 \times 10^{3} \text{ m s}^{-1} = 7.61 \times 10^{3} \text{ m s}^{-1} \text{ (3 sf)}$$