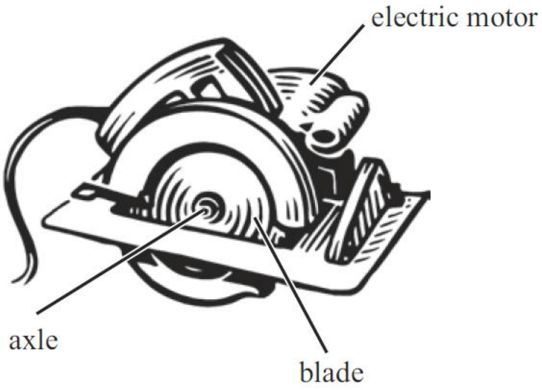
MECHANICS: ANGULAR MOMENTUM QUESTIONS

ROTATIONAL MOTION (2011;1)

Brad has a job at a construction site. He often uses power tools on the site. A hand-held circular saw contains a disc-shaped blade that spins rapidly to cut through wood. The axle is fixed through the centre of the blade. An electric motor makes the axle spin, and this in turn makes the blade spin.

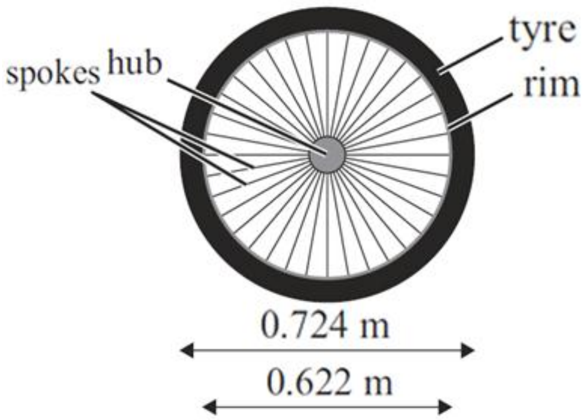


(a) When operated at full speed with no load, the blade rotates at 4 500 revolutions per minute. Show that the angular velocity of the blade is 471 rad s^{-1}.

(b) The rotational inertia of the blade is 1.28×10^{-3} kg m^2. It takes 0.12 s for the motor to accelerate the blade from rest up to full speed. Calculate the average power provided by the motor during this time.

ROTATIONAL MOTION (2010;1)

The bicycle wheel shown in the diagram consists of a set of lightweight spokes that radiate out from the hub to the rim. The wheel has a rotational inertia of 0.640 kgm^2. The rim diameter of the wheel is 0.622 m and the tyre diameter is 0.724 m.



(a) (i) The wheel rotates at a steady speed. It completes one revolution in 0.740 s. Calculate its angular velocity.
(ii) Calculate the rotational kinetic energy of the wheel.

(b) The rotational inertia of a hollow cylinder of mass *m*, and radius *r*, can be calculated from $I = mr^2$. Discuss how you could use this formula to calculate the rotational inertia of the wheel and what assumptions you would have to make.

(c) A cyclist is riding a bike at a speed of 5.50 ms^{-1}. Both wheels on the bicycle are the same as the one in the diagram. The bicycle and its rider have a total mass 68.4 kg. Calculate the work that was done to accelerate to this speed. You may ignore any frictional effects.

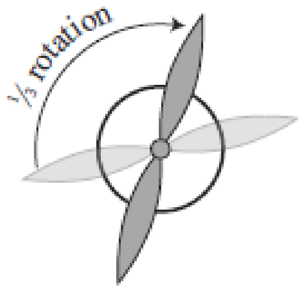
(d) Two common types of bicycle are road bikes and mountain bikes. Mountain bike wheels generally have a lower rotational inertia than road bike wheels. A mountain bike wheel and a road bike wheel are both accelerated at the same rate. Explain why it is easier to accelerate the mountain bike wheel than the road bike wheel.

QUESTION TWO (2009;2)

To start the plane, Sam has to turn the propeller by hand. When the propeller reaches a high enough angular velocity, the engine will be able to start. The propeller has a rotational inertia of 16.5 kgm^2. The engine will start when the propeller is turning at 100 revolutions per minute (rpm).

(a) Show that 100 rpm is 10.5 $rads^{-1}$

Sam is able to apply a constant torque to the propeller for 1/3rd of a revolution.



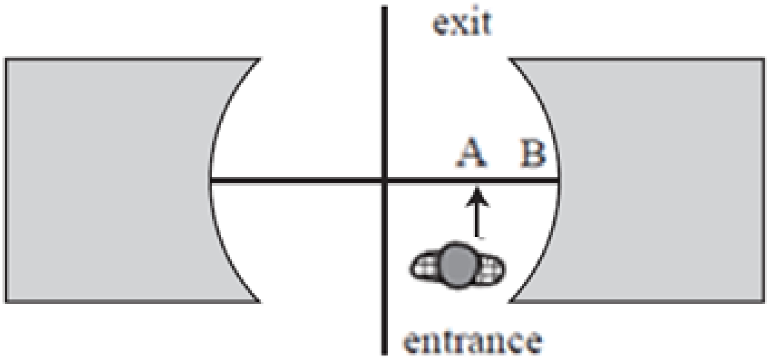
The propeller is at rest when Sam starts to turn it. Calculate the minimum constant torque he must apply to start the engine.

(b) Sam wants to fit a new propeller, of the same length, which will be easier to accelerate to 100 rpm. Explain how the new propeller might be different to the existing propeller.

When the engine is shut off, the propeller takes 20.0 s to come to a complete stop from an angular velocity of 37.7 $rads^{-1}$. Calculate the number of turns the propeller completes while it slows to a stop. State any assumption(s) you have made.

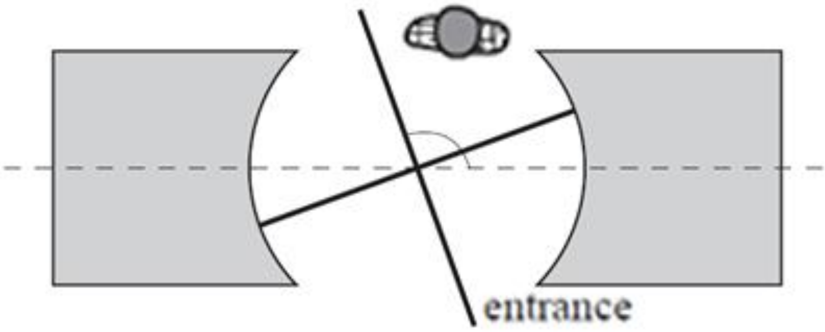
ROTATIONAL MOTION (2008;1)

Revolving doors are used in many big buildings. You may assume that the effects of friction can be ignored in this question.



Jenny enters a revolving door, which is initially stationary (above diagram). She pushes on the door at point A: it accelerates at 0.48 rads^{-2}. She stops pushing when it reaches an angular velocity of 0.58 rads^{-1}.

(a) Jenny pushes, at right angles to the door, with a force of 132 N at a point 83 cm from the central axis. Show that she exerts a torque of 110 Nm.
(b) Show that the rotational inertia of the door is 230 kgm^2.
(c) Calculate the rotational kinetic energy gained by the door from Jenny.
(d) Explain how this gain in energy is related to the force Jenny exerted on the door.
(e) Dorothy tells Jenny that by pushing at B she can get the door rotating to the same speed in the same time with less force. Discuss whether this idea is correct.



(f) The door has to rotate through a total angular displacement of 2.0 rad to allow Jenny to walk through the door (above diagram). This total includes the angular displacement during acceleration and the angular displacement at constant angular velocity. Calculate the angular displacement of the door, from the instant it reaches a constant angular velocity, until it has rotated through a total of 2.0 rad.
(g) Show that the total time that Jenny is inside the revolving door (from the moment she starts pushing until she exits), can be expressed as an equation:

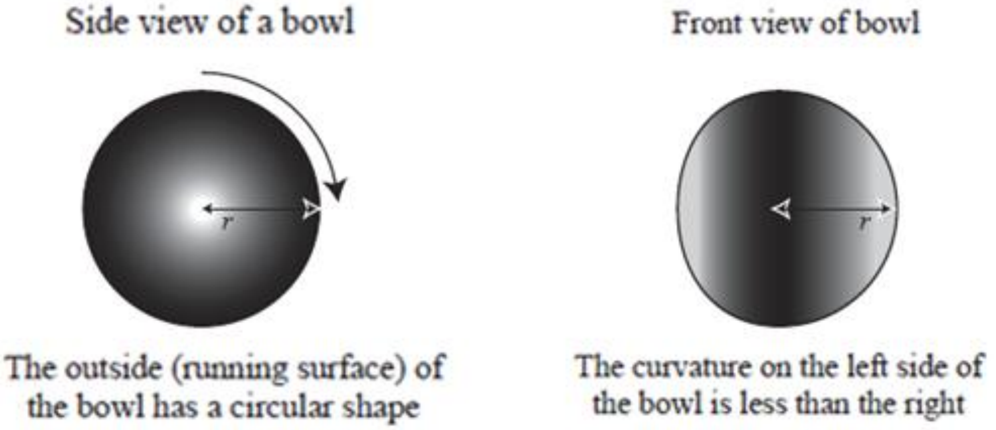
$$t_{\text{total}} = \frac{\omega}{\alpha} + \frac{\theta_2}{\omega}$$

where ω is the maximum angular velocity of the door, α is the angular acceleration of the door and θ the angle through which the door rotates from when she stops pushing until when she exits.

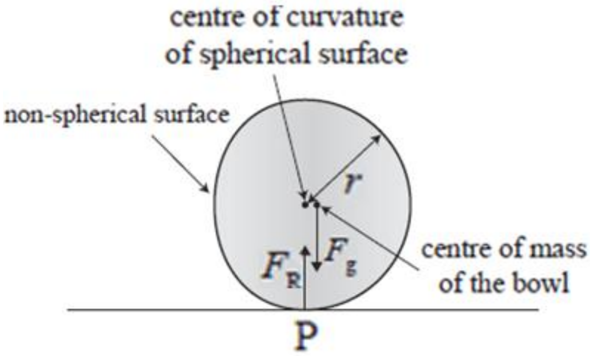
QUESTION TWO (2007;2)

Acceleration due to gravity = 9.81 ms^{-2}.

A slowly moving bowl will travel in a curved path, not a straight path. The curved path of the bowl is due to its slightly non-spherical shape, which is created by shaving a small amount off one side of the bowl. The diagrams below show the shape of the bowl.

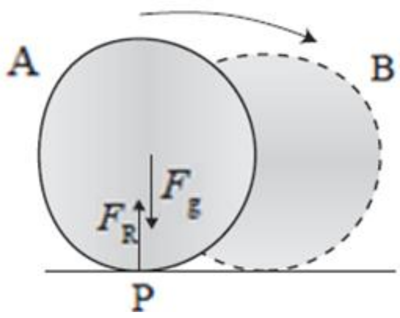


The diagram below shows the bowl, viewed from the front, held stationary on a flat surface. The reaction force, F_R, on the bowl acts from the point of contact, P, through the centre of curvature of the spherical part. The gravity force, F, acts through the centre of mass of the bowl.



(a) State why the centre of mass is not in the same position as the centre of curvature.

If the bowl is to stay in the position shown, it must be held there. If it is released it will roll over sideways, as shown in the diagram below, to position B.



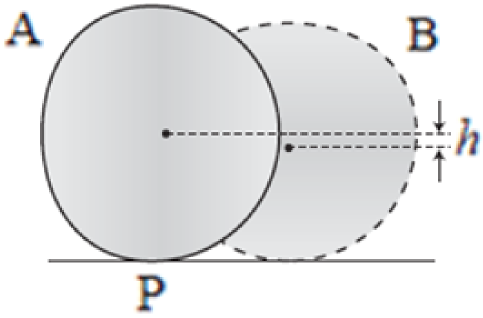
(b) Explain why, when it is released, the bowl will not stay balanced at position A.

When the bowl reaches position B, it has both rotational and linear kinetic energy. At position B, the bowl has a rotational speed of 2.8 rads $^{-1}$.

(c) The rotational inertia of the bowl is 2.16 x 10^{-3} kgm^2. Show that the rotational kinetic energy of the bowl at position B is 8.5 mJ.

(d) The mass of the bowl is 1.50 kg and the radius, r, of its spherical surface is 0.060 m. Show that the linear kinetic energy of the bowl when it reaches position B is 21 mJ (You may assume that the centre of mass travels in an approximately horizontal line).

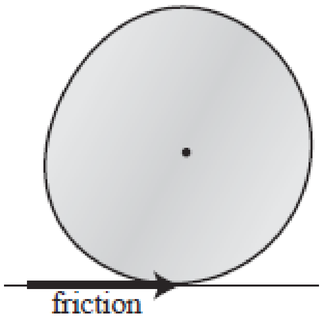
In fact, the centre of mass of the bowl drops slightly as shown in the diagram below.



(e) At position B, where has the linear and rotational energy come from?

(f) Calculate the distance the centre of mass of the bowl drops as it rolls from position A to position B (Ignore any energy losses).

While the bowl is rolling over sideways, friction between the bowl and the flat surface is acting in the direction shown below.



(g) Explain why friction acts in this direction.

(h) When the bowl is rolling forwards (after it has been bowled), this frictional force will still be acting sideways (at right angles to the forward direction). Explain why, after it has been bowled, the path of the bowl curves.

DIVING OFF THE HIGH BOARD (2006;3)

Hopi performs a dive from the high board. After B leaving the board at A, he travels up in the air to B, tucking his body into a ball. At the end of his dive, he straightens his body and enters the water head first at C. When Hopi's body is in the tucked position during the rotations, his rotational inertia is 3.73 kgm^2. Hopi's mass is 76 kg.

(a) When Hopi's body is in the tucked position, his shape can be modelled by a solid sphere of rotational inertia $I = \frac{2}{5}mr^2$

Calculate the radius of the sphere that models Hopi's shape. Explain why Hopi must tuck his body if the rotations are to be completed before he enters the water.

While his body is in the tucked position, Hopi's angular speed is a constant 9.82 rads^{-1}. He does two complete rotations in this tucked position.

(b) Show that his angular momentum, while he is rotating in the tucked position, is 36.6 kgm^{2}s^{-1}.

(c) Calculate the time it takes him to complete the two rotations in the tucked position.

At the end of the tucked rotations, Hopi straightens his body for the entry into the water.

(d) What physics principle applies while Hopi is straightening his body?

It takes Hopi 0.32 s to straighten his body, and during this time his rotational inertia increases by factor of 5.

(e) Calculate his angular deceleration during this time.

LINEAR AND ROTATIONAL MOTION (2005;1)

The London Eye is a giant rotating wheel that has 32 capsules attached at evenly spaced intervals to its outer rim. Passengers riding in the capsules get spectacular views over London, especially at the top.

Capsule

The capsules each have mass 1.0×10^4 kg and are at a distance of 68 m from the centre of the wheel. They travel at a constant speed of 0.26 ms^{-1}.

(a) Show that the angular speed of the wheel is 3.8×10^{-3} rads

(b) Calculate the time it takes the wheel to travel a complete revolution.

When the London Eye is started up each day there are no passengers in the capsules and it takes2.3 s for an average net torque of 4.6×10^7 Nm to accelerate the wheel from rest to its operational speed of 0.26 ms^{-1}.

(c) Calculate the average angular acceleration of the wheel.

(d) Calculate the angle, in degrees, the wheel turns through during start-up.

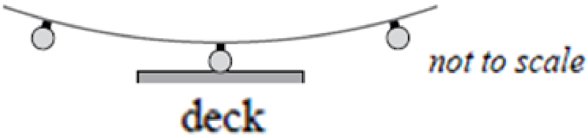
(e) Calculate the rotational inertia of the wheel.

(f) In practice, the angular acceleration will gradually reduce to zero during the 2.3 s. It will continue to be zero after this time even though a torque continues to be applied. Explain why the angular acceleration behaves in this way.

(g) At what point on the wheel could a force be applied to give maximum torque?

(h) By considering what happens to kinetic energy, gravitational potential energy and any other forms of energy, describe and explain the energy transformations that occur while the wheel rotates.

The passengers enter and leave the capsules from a deck at the bottom of the wheel. Because the wheel does not stop, each capsule is travelling in the arc of a circle as it moves past the horizontal deck.



It takes 29 s for a capsule to pass in front of the deck. The first part of the deck, which is passed during the first half of this time, is for passengers leaving the capsule. The second part of the deck is used by passengers entering the capsule.

(i) (i) Calculate the angle through which the wheel turns while one capsule is passing in front of the whole length of the deck.

(ii) Calculate the length of deck that is used by passengers entering the capsule.

(j) At the start of the day, when passengers are entering the capsules, will angular momentum of the wheel be conserved? Explain your answer. Assume the speed of the capsules stays constant.

A passenger is inside a moving capsule. At any position in the ride her motion can be considered to be linear (because she has a tangential velocity), or it can be considered to be rotational (because she is rotating about the centre of the wheel). Her kinetic energy, therefore, can be considered to be either linear or rotational.

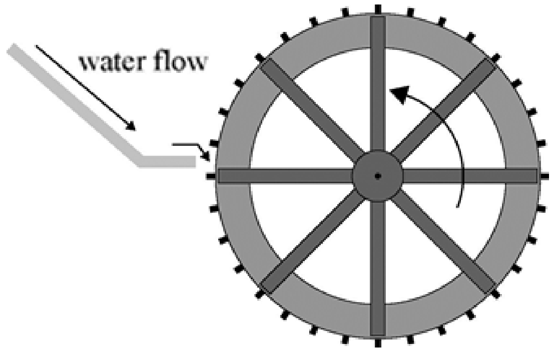
(k) Using this energy consideration, derive a formula for her rotational inertia.

The mass of the passenger is 65 kg

(l) Calculate the angular momentum of the passenger about the centre of rotation of the wheel.

JOHN BYCROFT'S WATERWHEEL (2004;1)

At Howick Historical Village in Auckland there is an old 19th century iron water wheel called the John Bycroft. It was used to generate power to turn grindstones that made flour for his famous biscuits. If you pay $1, you can activate a switch to turn on the water flow and see the wheel in action. A diagram of the waterwheel is shown.



When the water first turns the wheel, it accelerates. After some time the angular speed of the wheel reaches its maximum and then stays constant. At this constant angular speed, the wheel completes one revolution in 20.0 s.

(a) Show that the constant angular speed is 0.314 $rads^{-1}$

The rotational inertia of the wheel about its axis of rotation is 3.50×10^4 kgm^2.

(b) Calculate the rotational kinetic energy of the wheel at the constant angular speed.

(c) (i) Calculate the angular momentum of the wheel about its axis of rotation, at this constant angular speed.

(ii) During the time the wheel is rotating at this angular speed, is angular momentum conserved? Give a reason for your answer.

(d) When the water first turns the wheel, it accelerates to an angular speed of 0.200 $rads^{-1}$ in 17.2 s. Show that the angular acceleration is 0.0116 $rads^{-2}$

(e) The radius of the wheel is 1.375 m and its rotational inertia is 3.50×10^4 kgm^2. Calculate the average force applied to the blades of the wheel by the water flow to make it accelerate.

(f) Some school students were doing a project to decide how the angular acceleration of the wheel could be increased. They suggested making the wheel out of wood instead of iron, but with the same dimensions. Assuming that the applied torque was the same, would this achieve their aim? Explain your answer.