In 2013, AS 91524 replaced AS 90521.

The Mess that is NCEA Assessment Schedules....

In AS 90521 there was an Evidence column with the correct answer and Achieved, Merit and Excellence columns explaining the required level of performance to get that grade. Each part of the question (row in the Assessment Schedule) contributed a single grade in either Criteria 1 (Explain stuff) or Criteria 2 (Solve stuff). From 2003 to 2008, the NCEA shaded columns that were not relevant to that question (Sorry haven’t had time to do 2004 yet).

In 91524, from 2013 onwards, each part of a question contributes to the overall Grade Score Marking of the question and there are no longer separate criteria. There is no shading anymore. There is no spoon. At least their equation editor has stopped displaying random characters over the units.

<table>
<thead>
<tr>
<th>Question</th>
<th>Evidence</th>
<th>Achievement</th>
<th>Merit</th>
<th>Excellence</th>
</tr>
</thead>
<tbody>
<tr>
<td>2017(3)</td>
<td>The direction of the restoring force does not change OR the direction of the restoring force remains down OR the restoring force increases as she moves away from equilibrium OR Accept equations in the explanation ( F = -ky ) or ( F \propto -y ) to this particular situation.</td>
<td>Correct answer.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(b) Correct diagram.

OR
Correct answer with insufficient working.

OR
Correct angle in either degrees or radians

OR
Used appropriate equation
e.g. \( y = A \sin \omega t \)

OR
Correct angular frequency
\( \omega = 0.785 \text{ rad s}^{-1} \)

\[
\begin{align*}
\text{In degrees:} & \\
\sin \theta &= 0.08 \\
\theta &= 41.8^\circ \\
t &= \frac{41.81^\circ \times 8.00}{360^\circ} = 0.928 \text{ s} \\
\end{align*}
\]

\[
\begin{align*}
\text{In Radians:} & \\
\sin \theta &= 0.08 \\
\theta &= 0.73 \text{ rad} \\
t &= \frac{\frac{\theta}{\omega} \times \frac{2\pi}{T}}{2\pi} = 0.73 \times 8.00 \\
t &= 0.929 \text{ s} \\
\end{align*}
\]

\[
\begin{align*}
y &= A \sin \omega t \text{ and } \omega = \frac{2\pi}{T} \text{ Note: } \omega = 0.785 \text{ rad s}^{-1} \\
0.08 &= 0.12 \sin \left( \frac{2\pi}{8.00} \right) \\
2\pi &= \sin^{-1} \left( \frac{0.08}{0.12} \right) \\
2\pi &= 0.7297 \text{ rad} \\
t &= \frac{0.7297 \times 8.00}{2\pi} \\
t &= 0.929 \text{ s} \\
\end{align*}
\]
Correct answer for $v_{\text{max}}$. OR
Graph correct shape.

Correct answer for $v_{\text{max}}$. AND
Graph correct shape.
This formula states that if the mass is increased, the period increases also. In order to get the true mass of the astronaut, the mass of the seat and the effective mass of the spring should be omitted. However, because this information is not known and because the question states that the seat is “lightweight”, the mass of the seat (and spring) can be omitted for simplicity.

OR

For using the energy conversion to find mass, candidate justifies WHY it is appropriate to equate kinetic energy to elastic potential energy, i.e. negligible heat losses so energy is conserved.
<table>
<thead>
<tr>
<th>2016(3)</th>
<th>The acceleration (or restoring force) is proportional to displacement and is (acts) in the opposite direction to displacement. Accept equations in the explanation e.g. ((a = -\omega^2 y)), ((a \propto -y)), ((F = -ky)) or ((F \propto -y)). Both points stated.</th>
</tr>
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<tr>
<td>(a)</td>
<td>The acceleration (or restoring force) is proportional to displacement and is (acts) in the opposite direction to displacement. Accept equations in the explanation e.g. ((a = -\omega^2 y)), ((a \propto -y)), ((F = -ky)) or ((F \propto -y)). Both points stated.</td>
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<tr>
<td>(b)</td>
<td>When (T = 1.57) s, (\theta = 360^\circ) When (t = 0.25) s, (\theta = 57.3^\circ) (a = A \omega^2 \cos \theta) (= 0.10 \times 4.00^2 \cos 57.3 = 0.864) m s(^{-2}) OR (a = 2\pi \frac{2\pi}{T} = 4.00) rad s(^{-1}) (= A \omega^2 \cos t) (= 0.10 \times 4^2 \cos (4 \times 0.25)) (= 0.864) m s(^{-2}) Correct angle for (t = 0.25) s. OR Correct value for angular frequency. OR Use of suitable equation (a = A \omega^2 \cos ) One error in calculation. e.g.: Forgot to square (\omega). OR Used 10.0cm instead of 0.10m. Correct calculation and correct answer for acceleration with unit. (E) Correct calculation and answer for acceleration without unit. (E-)</td>
</tr>
<tr>
<td>(c)</td>
<td>This is an example of resonance. Tom must put energy into the system (the driving frequency) at the natural frequency. The natural frequency is the frequency at which the bumble bee will normally oscillate. (Amplitude increases until energy input equals kinetic energy lost as heat.) Idea of resonance or that the frequency of the driver matches the natural frequency of the spring. OR “Resonance” stated but not explained. Idea of resonance including the word “Resonance”. AND Links matching driving frequency to natural frequency. OR Explains maximum amplitude.</td>
</tr>
</tbody>
</table>
(d) Correct damped shape starting from +y for 3 complete cycles.
OR
Correct damped shape for 3 complete cycles and constant period and at least one value on axes (\(A_0 = 20\) cm, and \(T = 1.57\) s).
OR
States assumptions of zero damping and the graph shows this 3 complete cycles.
Constant period and at least one value on axes (\(A_0 = 20\) cm, and \(T = 1.57\) s).

2015(2)

(a) The passenger and the bathroom scales are falling at the same rate, so there are no contact forces between them. Accept support/normal forces/reaction force and free falling.

(b) \(a = -1.54 \times 10^{-6} y, y = A\) and \(\omega^2 = 1.54 \times 10^{-6}\)
\(a_{max} = -A\omega^2 = 1.54 \times 10^{-6} \times 6.38 \times 10^6 = 9.83\) m s\(^{-2}\)
OR
Maximum acceleration at Earth’s surface is \(g = 9.81\) m s\(^{-2}\).

(c) \(w = 1.54 \times 10^{-6}\) rads\(^{-1}\)
\(V_{max} = A \times \omega = 6.38 \times 10^6 \times 1.24 \times 10^{-3} = 7910\) m s\(^{-1}\)
OR
\(v = -\omega^2 \sin \omega t\)
\(V_{max} = A \times \omega = 6.38 \times 10^6 \times 1.24 \times 10^{-3} = 7910\) m s\(^{-1}\)
Combination of $f = \frac{1}{T}$ and $= 2\pi f$ to get $T = \frac{2\pi}{f}$

OR

Use of $\frac{2\pi}{f}$ and

$\omega^2 = 1.54 \times 10^{-6}$ to get 5063 s (84 minutes).

The journey time is 42 minutes (half the time period).

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$\omega^2 = 1.54 \times 10^{-6}$ to get 5063 s (84 minutes).

The journey time is 42 minutes (half the time period).
### 2014(2)

<table>
<thead>
<tr>
<th>(a)</th>
<th>( T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{1.55}{9.81}} = 2.50 \text{ s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t = \frac{1}{2}T = 1.25 \text{ s} )</td>
</tr>
</tbody>
</table>

### 2013(3)

<table>
<thead>
<tr>
<th>(a)</th>
<th>SHOW THAT QUESTION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{1.2}{9.81}} = 2.1975 = 2.2 \text{ s} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b)</th>
<th>Angle of the swing must be small, so that the motion can be approximated as a straight line and the force is proportional to the displacement.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Angle / amplitude must be kept small. The motion can be approximated to a straight line OR force / acceleration are proportional to displacement OR force / acceleration is towards the equilibrium position.</td>
</tr>
<tr>
<td></td>
<td>Angle / amplitude / initial displacement is small AND (The motion of the ball can be approximated to a straight line OR force / acceleration is proportional to displacement OR force / acceleration is towards the equilibrium position).</td>
</tr>
</tbody>
</table>

| (c) | ![Energy vs Time Diagram](image) |
|-----| Downwards sloping line. Oscillation with constant period that decreases with time. |
|     | Line has negative slope with a decreasing size of slope. |
(d) The ball will start oscillating if the frequency of the shaking matches the natural frequency of the pendulum. If the frequencies are the same, resonance will occur, which means the energy used for the shaking will be transferred into the ball, giving it kinetic energy to make the ball swing.

| Shake at (resonant frequency OR natural frequency OR same time each cycle). $E_{\text{grav}} \leq E_k$ as ball oscillates. Standing wave set up (or words describing this). | Shaking at (resonant frequency OR natural frequency OR same time each cycle). AND will cause (large amplitude OR large energy transfer OR energy transferred from shake to ball). Wave reflects / interferes and standing wave is set up. Shake at (resonant freq OR natural freq OR same time each cycle) AND ($E_{\text{grav}} \geq E_k$ as ball oscillates OR statement of conservation of energy). Force of hand pulls ball (increasing speed / acc / restoring force / amplitude OR causing acceleration / restoring force). | Shaking at (resonant frequency OR natural frequency OR same time each cycle). AND Causes (energy to be transferred from shake to $E_{\text{grav}}$ or $E_k$ of ball OR wave reflects and interferes producing standing wave). |
The forces acting on the ball are the tension force and the gravitational force. The gravitational force is constant, and always acts downwards. The tension force changes direction as the ball swings because it always points in the direction of the cord.

At each end of the swing, the tension plus the gravitational force add to make the net force which is the restoring force which is at a tangent to the path. The tension force balances the component of the gravitational force opposite to the direction of the string. The restoring force is the component of the gravitational force that is perpendicular to the string. As the displacement increases, the angle of the string increases, so the size of the restoring force increases.

Note: If students use the word “support” in place of tension, drop mark down by one.

Gravitational force and tension identified as forces acting on the ball. (Symbols sufficient) Gravitational force does not change

Gravitational force and tension named and labelled diagram of tension at correct non-zero angle and gravitational force acting downwards. No incorrect forces (e.g. \( F_c \)).

Tension increases displacement decreases because the centripetal force is greater because it is moving faster (curved)

OR

Tension increases as displacement decreases as tension force cancels a component of gravity (curved)

OR

Tension decreases as displacement decreases as horizontal component of tension gets smaller (straight line) OR

Tension decreases as displacement decreases as vertical component cancels gravitational force (straight line)

OR

Tension changes angle as it always acts towards the cord

(Restoring force is component of (tension or gravitational force) AND restoring force increases with greater angle)

Note: Accept air resistance included.
<table>
<thead>
<tr>
<th>2012(2)</th>
<th>a = −ω^2 y ⇒ a = 1.45^2 × 0.80 = 1.682 = 1.68 m s^{-2}</th>
<th>^2Correct answer.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
<td>1Correct equation selected.</td>
<td>^2Correct displacement value</td>
</tr>
</tbody>
</table>
| (b) | y = A cos ωt  
= 0.80 × cos (1.45 × 1.8) = −0.6896  
The negative means Daniel's displacement from the equilibrium position is opposite to when he started, which means he has gone through the equilibrium position.  
⇒ total distance = A + y = 0.80 + 0.6896  
= 1.4896 = 1.49 m | | ^2Correct distance |
| (c) | If Daniel had pushed off, his speed as he started his swing would not have been zero and so he would have had kinetic energy. His kinetic energy increase as he drops to the lowest position in his swing would be the same and so his maximum kinetic energy would be greater. This means he would rise further before his kinetic energy has been changed to gravitational potential energy, thus increasing the amplitude of his motion.  
As a swinging pendulum approximates to simple harmonic motion only if the amplitude is small, increasing the amplitude may mean the motion is no longer simple harmonic. | ^3Explanation shows some idea of: increase in initial (angular) speed. OR Some increase in energy. OR Increased amplitude means no longer SHM. | ^2Correct explanation for why the energy has increased. OR Correct explanation for why increase in energy means increase in amplitude. OR Correct explanation for why increased amplitude means no longer simple harmonic motion. | ^2Correct explanation for why the energy has increased and why increase in energy means increase in amplitude. OR Correct explanation for why the energy has increased and correct explanation for why increased amplitude means no longer SHM. |
(d) The period of the pendulum depends on its length \( (T = 2\pi \sqrt{L/g}) \). The length of a pendulum is from the pivot point at the top to the centre of mass of the bob, which is Daniel. If Daniel is sitting down his centre of mass will be further down and so the length of the pendulum will be greater and so the period will be longer.

<table>
<thead>
<tr>
<th>Idea that the period depends on length and that the length will change.</th>
<th>Correct and full explanation.</th>
</tr>
</thead>
</table>

### 2011(2)

#### (a)

\[
T = 2\pi \sqrt{\frac{L}{g}}
\]

- \( T = 2\pi \sqrt{\frac{5.50}{9.81}} \)
- \( T = 4.7046 \text{ s} \)
- \( T = 4.70 \text{ s} \)

Correct working.
This is a show question.

#### (b)(i)

\[
\begin{align*}
v_{\text{max}} &= A \\
&= \frac{2}{4.70} \\
&= 1.337 \text{ rad s}^{-1}
\end{align*}
\]

\[
\begin{align*}
v_{\text{max}} &= A = 0.90 \times 1.337 \\
&= 1.20 \text{ m s}^{-1}
\end{align*}
\]

Correct \( \omega \).
Correct answer.

#### (ii)

Equilibrium position shown.

Correct.
(c) At all points the tension force has to provide the centripetal force required to keep the bag moving in a circle and balance a component of the force of gravity. At the equilibrium point, the tension is greatest because the speed is greatest and the gravity component is the full gravity force. At the extremes, the speed is zero, so there is no centripetal force, and as one component of the gravity force is providing the restoring force the other component, which must be balanced by the tension force is therefore less than the full gravity force. For both these reasons the tension force at the end positions has decreased.

\[ \text{mg} \]
\[ \text{tension} \]

| Recognition that the tension force provides the centripetal force. |
| Recognition that tension force must provide both the centripetal force and must balance gravity. |
| Explanation for why the centripetal force is greatest at the equilibrium position. |
| OR |
| Explanation for why the gravity component that tension must balance is greatest at the equilibrium position. |

(d) \[ T = 2\pi \sqrt{\frac{l}{g}} = \frac{2\pi}{\omega} \]
\[ \omega = \frac{v}{r} \propto \sqrt{\frac{1}{l}} \]

So increasing length makes \( v_{\text{max}} \) decrease.

<p>| ( \omega ) decreases because the time period is increases. |
| ( v ) decreases because ( T ) increases and angular frequency decreases. |</p>
<table>
<thead>
<tr>
<th>Question</th>
<th>Correct Description</th>
<th>Correct Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e)</td>
<td>( \theta_{\text{tot}} = \sin^{-1} \frac{0.8}{0.9} + \sin^{-1} \frac{0.5}{0.9} ) [= 1.68 \text{ rad} ] ( t = \frac{\theta_{\text{tot}}}{\omega} = 1.68 \left( \frac{4.70}{2\pi} \right) ) [= 1.26 \text{ s} ]</td>
<td>( \Rightarrow n = \frac{83.8}{2.61799} = 32.01 ) [= 32 \text{ oscillations.} ]</td>
</tr>
<tr>
<td>(2010(3))</td>
<td>( a = -\omega^2 y ), Size of the acceleration must be proportional to the size of the displacement.</td>
<td>( \Rightarrow ) Correct description.</td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td>( \Rightarrow ) Correct T</td>
</tr>
<tr>
<td>2010(3)</td>
<td>Friction works against the motion, changing energy to heat. The amplitude depends on the energy of the motion, so as energy is lost, the amplitude decreases.</td>
<td>( \Rightarrow ) Decrease in amplitude linked to decrease in energy.</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td>( \Rightarrow ) Decrease in amplitude linked to decrease in energy which is linked to the action of friction changing energy to heat etc.</td>
</tr>
<tr>
<td>(c)</td>
<td></td>
<td>( \Rightarrow ) Correct answer.</td>
</tr>
</tbody>
</table>
### No Brain Too Small ⚫ PHYSICS ⚫

#### (d)(i)

\[ F = ma = -m\omega^2 y = 1.65 \times 2.40^2 \times 0.0270 \]
\[ = 0.2566 \text{ N} \]

Correct value for \( a \).

**OR**

Correct answer.

Correct working and answer.

#### (ii)

\[ v = A\omega \cos \omega t, \ y = A\sin \omega t \]

\[ \Rightarrow \omega t = \sin^{-1} \frac{y}{A} = \sin^{-1} \frac{0.027}{0.147} = 0.18472 \]

\[ \Rightarrow v = 0.147 \times 2.62 \times \cos[0.183673] \]
\[ = 0.34680 \text{ m s}^{-1} \]

Correct \( \omega t \) or \( t \).

Correct answer consistent with using \( y \) in centimetres.

Correct working and answer.

#### 2009(3)

(a)

THIS IS A SHOW QUESTION

\[ k = F/x \]
\[ F = mg = 635 \times 9.81 \text{ N} \]

\[ k = 635 \times 9.81 / (0.175) \]
\[ = 35600 \text{ N m}^{-1} \]

Extension = 17.5cm and Force = 6229N

Correct working.

**OR**

Finds \( F \) or \( x \) and substitution OK x in m

(b)

THIS IS A SHOW QUESTION

\[ T = 2\pi \sqrt{\frac{m}{k}} \]
\[ = 2\pi \times \sqrt{\frac{635}{35600}} = 0.839 \text{ s} \]

Correct working.

\[ T = 0.8392 \text{ s} \]
\[ = 0.8391979 \text{ s} \]
| (c) | ![Diagram](image1.png) | \(^1\)Correct orientation of phasor.  
OR  
Any description of the planes velocity at this position.  
This could be a calculation or a description.  
v(shm) = 0.56 \text{ ms}^{-1} |  |
|-----|-----------------|-----------------|-----------------|
| (d) | \(a_{\text{max}} = A \omega^2\)  
\(\omega = 2\pi/T = 2\pi/0.839 = 7.49 \text{ rad s}^{-1}\)  
\(a_{\text{max}} = 0.15 \times 7.492 = 8.41 \text{ m s}^{-2}\)  
Vertical component at the time shown  
\(a = a_{\text{max}} \times \cos 30^\circ = -7.28 \text{ m s}^{-2}\). | \(^2\)Correct answer  
a = 7.28 \text{ m s}^{-2}. |  |
The shock absorbers reduce the amplitude of the SHM so the displacement-time and acceleration-time graphs would show a gradual decrease in amplitude (period and \( \omega \) is (roughly) constant) so

\[
a = -\omega^2 y
\]

\[
a \propto -y
\]

The discomfort of passengers is due to acceleration (constant velocity would not be noticeable).

### Describe the effect of moderate damping on:
- displacement
- acceleration

how it affects discomfort

1. reduced displacement
2. reduced acceleration

Describe the effect of moderate damping on:

Discomfort is proportional to acceleration or net force

OR

1. reduced displacement

OR

Graph of \( d \) vs \( t \)
vertical axis labelled or title

AND

reduced acceleration

OR

Graph of \( a \) vs \( t \)

why acceleration is reduced:

\[
a \propto (-)y
\]

### 2008(3) (a)

\[
T = 2\pi \sqrt{\frac{7}{g}}
\]

\[
T = 2\pi \sqrt{\frac{2.1}{9.81}} = 2\pi \times 0.46291 = 2.9070 = 2.9 \text{ s}
\]

A **SHOW** question

2. Correct substitution into correct equation
**2007(3)**  

(a) \[ T = \frac{9.32}{13} = 0.716923 = 0.717 \text{ s} \]  
Correct answer  
Answer given to 3 sf and FOUR answers given with correct units.

(b) \[ T = 2\pi \sqrt{\frac{R}{g}} \Rightarrow R = \frac{0.716923^2 \times 9.81}{(2\pi)^2} \]  
Correct answer consequential to answer given in 3(a).

(c) The restoring force is proportional to the displacement / in the opposite direction to the displacement / acts towards the equilibrium position.  
One statement is given.

(d) \[ F_R = \frac{F_y}{R}, \text{ and } F_R = ma \]  
Correct answer shows recognition that \( F_R \) also equals \( ma \).

but \( a = \omega^2 y = \left(\frac{2\pi}{T}\right)^2 \) and \( F_g = mg \)  
\[ \Rightarrow \frac{mg\omega^2 y}{R} = my\left(\frac{2\pi}{T}\right)^2 \]  
\[ \Rightarrow \ T = 2\pi \sqrt{\frac{R}{g}} \]  
Correct working

(e) Friction with the air and against the surface.  
Must state friction

(f) Because of the changed shape of the surface the forces opposing the bowl’s motion will be greater. This means that the rate at which the SHM energy is dissipated will be greater.  
Idea of increased forces acting against the bowl / idea of greater rate of change of energy.  
Idea of increased forces creating more energy change / of how a soft surface causes a greater rate of change of energy.  
Increased opposing forces clearly linked to greater rate of change of energy.
### 2006(1)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(a)</strong></td>
<td>(\omega = 2\pi f)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\Rightarrow f = \frac{2.2}{2\pi} = 0.35014 \text{ s}^{-1} \text{ (Hz)})</td>
<td>(^2\text{Correct answer. Has to be a calculation with } \omega.)</td>
</tr>
<tr>
<td><strong>(b)</strong></td>
<td>(T = 2\pi \sqrt{\frac{l}{g}})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.9 = 2 \times \pi \times \sqrt{\left(\frac{l}{9.81}\right)})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\Rightarrow l = 2.08980 \text{ m})</td>
<td>(^2\text{Correct answer.})</td>
</tr>
<tr>
<td><strong>(c)</strong></td>
<td>As the length is shortened, (T) will also decrease. If (L) is halved, (T) will decrease by square root of 2. (T = 2.056 \text{ s})</td>
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<td></td>
<td>(^1\text{Idea that } T \text{ will decrease if } L \text{ decreases.})</td>
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<tr>
<td></td>
<td>(^1\text{Idea that } T \text{ will decrease if } L \text{ decreases, and the appropriate factor is given. Calculation of } T \text{ can be included with the explanation.})</td>
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<tr>
<td><strong>(d)</strong></td>
<td>(a_{\text{max}} = \pm A \omega^2)</td>
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<td></td>
<td>(0.37 \times 2.2^2 = 1.7908)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a_{\text{max}} = 1.8 \text{ m s}^{-2})</td>
<td>(^2\text{Correct answer.})</td>
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<tr>
<td><strong>(e)</strong></td>
<td>Arrow drawn at the tangent to the path in the direction of the equilibrium position.</td>
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<td></td>
<td>(^1\text{Arrow drawn in correct direction.})</td>
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<td></td>
<td>(^2\text{Arrow drawn on tangent in correct direction.})</td>
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<tr>
<td></td>
<td>(f) [ F = mω^2 y = 31 \times 2.2^2 \times 0.25 = 37.51 , \text{N} ]</td>
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<td>(g)</td>
<td>[ \theta = ωt ] [ \cos ωt = \frac{37 - 12}{37} ] [ → ωt = 0.828915 \quad (47.5^\circ) ] [ → t = 0.377 , \text{s} ] [ t_{\text{total}} = t + \frac{1}{2}T ] [ = 1.826 = 1.8 , \text{s} ]</td>
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<tr>
<td></td>
<td>²Correct answer. ²Rounded to 2 sig. fig. plus 6 answers given with correct unit. ²Correct answer. ²Correct statement.</td>
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</tbody>
</table>
| (d) | As the timing starts from the end position the “sine” form of the formula must be used.  
\[ v = A \cos \omega t \]  
\[ = 1.5 \times 1.8 \times \sin(1.8 \times 0.75) \]  
\[ = 2.63445 \]  
\[ 2.6 \text{ m/s}^{-1} \] | Recognition that the “sine” form of the formula must be used. | Correct answer consistent with calculator in degree mode. | Correct answer |
| (e) | \( a = \omega^2 y \). This shows that the acceleration depends only on the displacement and angular speed. As the natural frequency is the same for both damped and undamped motion, the angular speed will stay the same. | One correct and relevant statement: \( a \) is proportional to \( y / \omega \) is constant. | Recognition that acceleration depends on displacement only. | Link made between the acceleration depending on displacement and angular speed only, and constant angular speed implies \( \omega \) depends on \( y \) only. |
| (f) | \[ T = \frac{2\pi}{\omega} = \frac{2\pi}{1.8} = 3.49066 \]  
\[ 3.5 \text{ s} \] | | Correct answer |
Every $\frac{1}{4}$ cycle the displacement phasor rotates through angle $\theta$ while the displacement is outside the maximum damped displacement.

$$\cos \theta = \frac{1}{2} \times 0.150$$

$$= \frac{1}{2} \times 3.0$$

$$= 87.134^\circ \quad (1.52078 \text{ rad})$$

total angle turned by displacement phasor in a complete cycle $= 4 \times \theta = 348.536^\circ$ (6.08310)

$$t = \frac{4\theta}{360} \times T \quad (t = \frac{4\theta}{\omega})$$

$$t = 3.37950 \quad (t = 3.3795) = 3.4 \text{ s}$$

Correctly calculated $\theta$.

Correct answer.