NZEST
ACADEMIC ACHIEVEMENT BY CHOICE

NEW ZEALAND
EDUCATION & SCHOLARSHIP TRUST

*Sponsored by:*

*ANZ Bank*
*Brierley Investments Ltd*
*CS First Boston NZ*
*Electricity Corporation of NZ*
*Fay Richwhite*
*Freightways Holdings Ltd*
*Jackson Russell Dignan Armstrong*
*Lion Nathan*
*Logan Campbell Trust*
*NZI Life*
*Russell McVeagh McKenzie Bartleet & Co.*
*Woolf Fisher Trust*

# NZEST SCHOLARSHIP EXAMINATION 1993

# PHYSICS

Time allowed: THREE hours

Ten minutes extra are allowed for reading this paper

INSTRUCTIONS TO CANDIDATES

1. The paper consists of 10 questions. The marks for each question are not equal and vary from 10 to 15. The total marks aggregate to 120.

2. You should attempt as many questions as you can in the available time. Although it is possible to answer all questions it is not expected that candidates will do so. Candidates are advised to invest their time in proportion to the marks indicated.

3. The examiner will give more credit to fully attempted answers to complete questions rather than partial attempts to several questions. Furthermore, correct numerical answers without adequate explanations will not necessarily receive full marks.

DATA WHICH MAY BE REQUIRED

| | | | |
|---|---|---|---|
| Acceleration of gravity | $g$ | = | $9.80\ \text{m s}^{-2}$ |
| Permittivity of free space | $\varepsilon_o$ | = | $8.85 \times 10^{-12}\ \text{F m}^{-1}$ |
| Permeability of free space | $\mu_o$ | = | $4\pi \times 10^{-7}\ \text{H m}^{-1}$ |
| Electronic charge | $e$ | = | $1.60 \times 10^{-19}\ \text{C}$ |
| Planck's constant | $h$ | = | $6.63 \times 10^{-34}\ \text{J s}$ |
| Speed of light | $c$ | = | $3.00 \times 10^{8}\ \text{m s}^{-1}$ |

**Question 1** *[12 marks]*

An enlightened physics student determines the period, $T$, of a simple pendulum by measuring repeated timings of 10 oscillations. She obtains an average value for 10 oscillations along with its associated uncertainty of

$$22.39 \pm 0.02 \text{ sec.}$$

The purpose of measuring $T$ is to determine the acceleration of gravity, $g$, from the well-known relation for the simple pendulum,

$$T = 2\pi\sqrt{\frac{\ell}{g}}.$$

The effective length, $\ell$, of this simple pendulum is known from corrected measurements to be

$$1.246 \pm 0.001 \text{ metre.}$$

(a) Determine from these measurements the value for $g$ and its absolute uncertainty. [6 marks]

The student, anxious to improve the accuracy of the value obtained for $g$, sets about measuring further sets of 10 oscillations but is distracted by a fellow student after commencing the counting/timing for a new set of data. The distraction is protracted, lasting some ten or so minutes. She notices the pendulum is still oscillating when she returns to her task and decides to stop the running stopwatch at the end of a complete oscillation. It then suddenly occurs to her that she can make use of this data to determine a more accurate value for the period of the pendulum, and hence $g$. The time recorded on her stopwatch for this measurement of an uncounted number of oscillations is

$$12 \text{ min } 46.24 \text{ sec.}$$

She also knows from her previous measurements that the uncertainty in this measurement is $\pm 0.04$ sec. Determine:

(b) the new, more accurate value for $T$ from this data. [4 marks]

(c) the new value for $g$ and its absolute uncertainty. [2 marks]

**Question 2** ***[10 marks]***

In the rocket propulsion of a space vehicle, a mass of high velocity gases is ejected from the vehicle at a constant velocity relative to the rocket engine, $V_e$, determined by the combustion characteristics of the engine. By considering a short enough time interval, $\Delta t$, during which a small mass of gases, $\Delta m$, is ejected from the rocket engine; it can be shown that the remaining mass of the vehicle, $M$, receives a compensatory boost in velocity relative to a stationary observer, $\Delta v$, such that

$$M\Delta v = V_e \Delta m \, .$$

(a) Describe the physical law that leads to the above equation and the system to which it applies.
[2 marks]

(b) Use the above equation to derive the following expression for the thrust force, $F$, exerted on the rocket by the exhaust gases,

$$F = V_e \mu \, ,$$

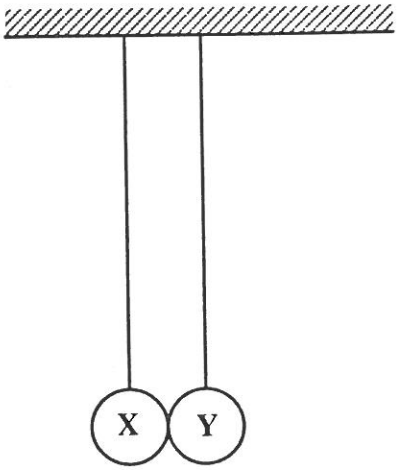
where $\mu$ is the rate of fuel consumption by the rocket. [3 marks]

(c) The first stage of a Saturn V space vehicle consumes fuel at the constant rate of $15 \times 10^3$ kg s$^{-1}$ and the gases have an exhaust velocity of $2.6 \times 10^3$ m s$^{-1}$. If the initial mass of the vehicle is $3 \times 10^6$ kg on the launch pad, find its **initial** acceleration off the launch pad.
[3 marks]

(d) Consider the motion of the rocket during the first 10 sec following lift-off. Assume it rises vertically upwards. Estimate the approximate speed and altitude attained by the vehicle 10 sec after launch.
[2 marks]

**Question 3** *[12 marks]*

The diagram shows two simple pendulums of equal length, 1.00 metre. Their bobs (X and Y) are just touching when the strings are parallel. The mass of X is 200 g and that of Y is 100 g.




(a) Y is drawn aside until the string makes an angle of 60° with the vertical and it is held in this position by a horizontal force. Find the magnitude of this horizontal force.

[2 marks]

(b) Y is released and swings to make a direct impact on X. In doing so, Y is brought to rest. Determine the height to which X rises.

[4 marks]

(c) Examine and compare the kinetic energies of the bobs before and after the collision just described in part (b). Discuss the energy transformations associated with the loss of kinetic energy in the impact.

[3 marks]

(d) The bob X, on returning to its initial position, strikes Y. Is X brought to rest at this impact? State yes or no and give your reasoning.

[3 marks]

**Question 4** *[13 marks]*

The following data may be required in answering the three questions asked below:

Speed of sound in air = 340 $m\,s^{-1}$
Speed of light = $3 \times 10^8$ $m\,s^{-1}$

(a) A student, studying in his flat, is listening to a live radio broadcast of a major rugby match when a fierce electrical thunderstorm occurs in the district. The student's flat is situated 2.0 km north of the rugby stadium across an extended region of flat terrain. A lightning bolt occurs at a position directly above the imaginary line joining the stadium to the student flat. The student first hears a noise generated by the electromagnetic pulse which accompanies the lightning bolt over his radio. Two and one half seconds later he hears over the radio the corresponding thunder picked up by the microphone in the broadcasting box of the stadium. Five seconds after he has heard the noise of the electromagnetic pulse on his radio, the thunder associated with the bolt rattles the windows of his flat. Where, relative to his flat, did the lightning bolt occur?

[5 marks]

(b) A national marching festival is being televised to a national audience. For one event a column of precision marchers decides to keep in step by listening to a band that is positioned at the head of their column. Throughout the event the music beats at 100 paces per minute. Much to the puzzlement of some observant television viewers, a zoom camera shot of the column shows the marchers at the back of the column to be a half-step out of step with the marchers at the front! (i.e. The marchers at the front are halfway through the pace with their left foot when the marchers at the back are commencing the pace with their left foot). However the marchers are so well trained and disciplined that they are all certain they are in proper step with the music. Explain the source of the problem and calculate the length of the column.

[3 marks]

(c) A hi-fi boffin outlines a method for connecting stereo speakers to an amplifier correctly so that they are in phase.

> *"Connect both speakers to the amplifier outputs. Now play a monophonic record/program with the bass control turned up and the treble control turned down to enhance the frequencies in the 60 hertz region. Listen carefully to the intensity from the speakers. Turn the balance control on the amplifier so that first one speaker is heard separately, then the two are heard together and finally the other speaker is heard separately. If the bass is stronger when both speakers play together, they are connected properly. If the bass is weaker when both play together than when each play separately, reverse the connections on one speaker."*

Explain why the procedure works, highlighting why a stereo source is not used and why only the bass is compared.

[5 marks]

**Question 5** *[15 marks]*

Read carefully the following report appearing in *New Scientist* of the earthquake that shook Mexico City on 19 September 1985, and then answer the questions asked below:

> *When the earthquake shook Mexico City on 19 September 1985, the worst-hit part of the city was an area that sits on the waterlogged sediments of an ancient lake. But within this area devastation was not uniform; buildings from 5 to 15 storeys high suffered the worst damage, and overall damage was distributed in alternate bands of heavy and light destruction.*
>
> *All objects from piano strings to bridges and tower blocks have a resonant frequency at which they vibrate naturally. The 2-second period of the incoming shocks coincided with the resonance frequencies of tower blocks between 5 and 15 storeys high, which explains their vulnerability. But the zonation of damage suggests that these shock waves were reflected internally within the basin, interfered with each other and gave rise to standing waves ('Nature', vol 326, p 783).*
>
> *Tremors are a complex mixture of up-and-down or side-to-side "shear" waves and back-and-fore "pressure" waves. Only the pressure waves move well through fluids or semi-solids. So the destruction in the region of the lake bed must have been caused by pressure waves. These waves originated locally, at the bottom of the sediment-filled basin, where about 30 percent of the energy from incoming shear waves would have been converted into pressure waves. Seismic data suggest the wavelength of these pressure waves was 3 kilometres.*
>
> *As the zones of maximum and minimum destruction were 750 metres apart, the observations fit the seismological prediction perfectly. Maximum collapses should be at the peak and trough of each wave, with minima at each end of the wave and at its central point, where displacement would have been at its lowest.*
>
> *Researchers at the Instituto de Fisica in Mexico City now want to refine their model to determine more precisely the nature and direction of movement at every locality within the boundary of the lake. This should help engineers to ensure that new "earthquake proof" buildings stand up rather better than their predecessors did.*

(a) State which waves in the article are longitudinal and which are transverse, and explain why. [3 marks]

(b) What was the resonant frequency of the tower blocks that proved most vulnerable to the quake? Indicate how you arrive at this conclusion. [2 marks]

(c) Using the seismic data offered in the third paragraph, explain why, in the fourth paragraph, the author is able to authoritatively assert that the 750 metre spacing of the maximum and minimum destruction zones *'fit the seismological prediction perfectly'*. [4 marks]

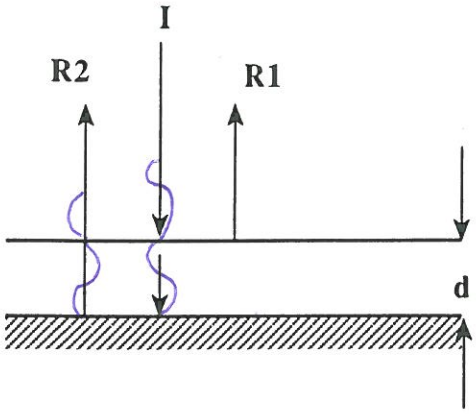
(d) Determine the velocity of a travelling seismic wave in the sediment-filled basin. [1 mark ]

(e) What kind of standing waves would you expect to have been set up in the tower block buildings - transverse or longitudinal? Give reasons. [2 marks]

(f) Given that one storey has a height of 3 metres, estimate the speed of travelling waves in the tower blocks that were damaged. [3 marks]

**Question 6** *[10 marks]*

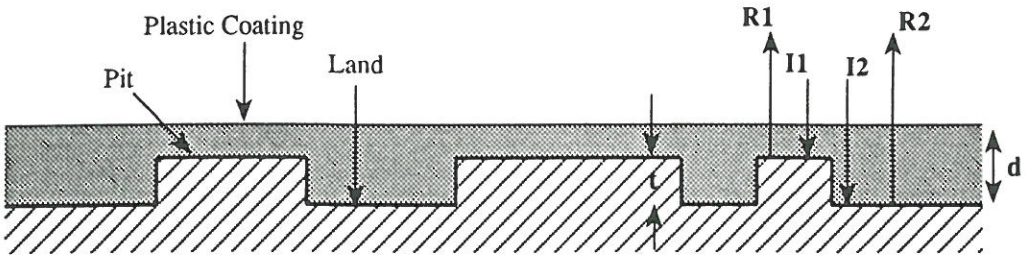
(a) The diagram immediately below shows monochromatic light of wavelength $\lambda$ falling at normal incidence on a thin film of thickness $d$ and refractive index $n$ smeared across a reflecting surface. The incident ray *I* is partially reflected *(R1)* from the air-film interface and subsequently reflected *(R2)* from the lower film-reflector interface. Interference takes place between the reflected rays *R1* and *R2*.




Establish the relation that applies for the condition of destructive interference when rays *R1* and *R2* recombine.

[4 marks]

(b) A compact disc (CD) contains a spiral track that holds audio information detected in the reflected beam of a laser. The CD track consists of raised areas of thickness $t$ called *pits* that are separated by flat areas called *land*. Both pits and land are covered with a level, thin transparent plastic coating of refractive index $n$ and thickness $d$ in the land region. As the CD rotates, the reflected light intensity fluctuates (due to interference effects) as the pits and land areas of the track pass by the laser beam, conveying the audio information as a series of binary numbers.




The diagram above shows a longitudinal section of the track with two sections of the incident, narrow laser beam *(I1* and *I2)* reflecting from the raised pit *(R1)* and land *(R2)* at the edge of a pit. The thickness of the pits is chosen so that destructive interference occurs when the reflected beams *R1* and *R2* recombine.

(i) What special property of the laser beam ensures that interference effects can occur between reflected rays originating from **different** incident rays?

[2 marks]

(ii) Establish the relation that gives rise to the destructive interference described above, and determine the minimum thickness of the pits given the laser wavelength is 790 nm and the plastic coating has a refractive index of $n = 1.50$.

[4 marks]

**Question 7** *[12 marks]*

Capacitors have the physical property of capacitance, $C$, which depends upon $\varepsilon_o$, the permittivity of free space, and the geometry of their construction. In the case of the parallel plate capacitor, the area of the plates $A$ and their separation $d$, together with $\varepsilon_o$, determine $C$.

Capacitors are usually fabricated with an insulating dielectric material between their plates. If the material completely fills the gap between the plates and across their full area, the capacitance is enhanced by a factor $\kappa$, the dielectric constant for the material. If only part of the overall area of the plates of the capacitor is filled with dielectric then only that area of the capacitor has its capacitance enhanced.

(a) A common kind of computer keyboard is based on the detection of a change in capacitance. Each key is mounted on one end of a plunger, the other end being attached to a movable metal plate. When the key is depressed, the movable plate is pushed closer to the fixed plate and the capacitance increases. The electronic circuitry which senses that the key has been pressed can detect a change in capacitance of 1.0 pF. If the area of the metal plates is 95 $mm^2$ and the spacing between the plates, which is filled with a flexible material of uniform dielectric constant 3.50, is 4.00 mm before the key is depressed, determine how far the key must be depressed before the circuitry detects its depression.

[5 marks]

(b) A fuel gauge designed for use in a vehicle that runs on methanol uses a capacitor to determine the height of the fuel in its tank. The capacitance changes from its base value of $C$ when the tank is empty to the value of $\kappa C$, where $\kappa$ is the dielectric constant of methanol, when the tank is full. The capacitor consists of two rectangular plates of height $H$ and length $L$ that run parallel to a set of opposing sides of the rectangularly-shaped fuel tank. These plates are electrically isolated from the tank and are connected to electronic circuitry which can essentially determine the effective dielectric constant $\kappa'$ of the combined air and fuel between them. If the (variable) height of the fuel between the plates is $h$:

(i) derive an expression for the effective dielectric constant between the plates. [5 marks]

(ii) determine the value of the capacitance, relative to that when the tank is empty, for situations when the tank is one-quarter full, one-half full, three-quarters full and full, given $\kappa = 33$ for methanol.

[2 marks]

**Question 8** *[10 marks]*

A loudspeaker system has a permanent magnet machined in the E-shape shown in Fig. 1 of the diagram.

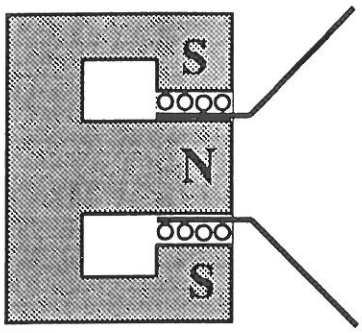



Fig. 1

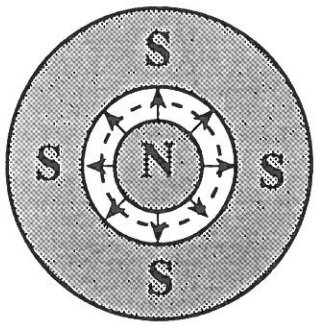



Fig. 2

The shape is designed to produce a uniform magnetic field $B$ in the annular gap, as shown in Fig. 2, into which the speaker coil is located. The magnitude of $B$ is 0.1 T.

A coil of radius $R$ = 1.0 cm and 1000 turns is wound on a thin cardboard cylinder which slides smoothly over the north pole of the magnet. The coil is connected to the loudspeaker cone.

(a) (i) If a steady current of 1.0 A flows in the coil, determine the force exerted on the speaker cone.

[2 marks]

(ii) Consider now an AC current of the form

$$I = I_o \sin \omega t ,$$

with a peak current $I_o$ = 1.0 A and a frequency of 500 Hz flowing in the coil. If the *effective* mass of the coil and cone is 150 g and the resistance of the air to its motion can be ignored, determine the amplitude of the ensuing SHM oscillations of the cone.

[3 marks]

(b) The loudspeaker can also be used as a microphone. Consider motion of the coil normal to the magnetic field. Convince yourself that when the coil is moved into the page with velocity $v$ any charge carriers within the wire of magnitude $q$ experience a tangential force $F$ in the clockwise sense of

$$F = qvB ,$$

producing a corresponding electric field given by

$$E = \frac{F}{q} = vB.$$

Use this result to:

(i) show that the emf, $\mathcal{E}$, generated in 1 turn of the coil due to $E$ is

$$\mathcal{E} = 2\pi RvB .$$ [1 mark ]

If a sound wave falling on the cone causes the coil to slide in SHM in the annular gap with a displacement $y$ given by

$$y = A \sin 2\pi ft ,$$

where the amplitude $A = 1 \times 10^{-6}$ metre and the frequency $f = 5 \times 10^3$ Hz

(ii) establish the expression for the emf induced in the coil as a function of time.

[4 marks]

**Question 9** *[12 marks]*

A metal detector used at an international airport consists of a series circuit formed by a real 2.50 mH inductor, a 2.00 μF capacitor and a generator with an RMS voltage of 10.00 volts. The inductor has the shape of a large rectangular coil, similar to a door frame of dimension 2.5 m x 1.0 m, that all boarding passengers are required to walk through. The frequency of the generator is held constant at the resonant frequency that applies when there is no metal passing through the coil. When a person with a metal object walks through the coil, the inductance increases and, consequently, the current in the circuit changes. The change in current can be then used to sound a warning. The resistance of the wire used to form the coil is 3.25 Ω and it is made from copper (resistivity $1.72 \times 10^{-8}$ Ωm) of radius 1.02 mm.

(a) Determine:

(i) the number of turns in the coil, [3 marks]

(ii) the RMS current when no metal is present, [1 mark ]

(iii) the frequency of the generator; and [2 marks]

(iv) sketch a phasor diagram showing the magnitudes of the RMS voltages across the coil, the capacitor and the generator. [3 marks]

(b) The RMS current changes to 2.70A when a metal object passing throught the detector causes a 5% increase in the inductance of the coil. Draw a new phasor diagram showing the relative RMS voltages across the capacitive, resistive and inductive elements in the circuit for this new situation. [2 marks]

(c) Estimate the number of kilowatt hours of energy consumed in a day when the detector is run continuously for 24 hours. [1 mark ]

**Question 10** *[14 marks]*

Radioactivity can be artificially induced by the absorption of low energy neutrons in otherwise stable isotopes, or by nuclear fission processes also induced by neutrons in fissile materials like uranium -235.

(a) Low energy neutrons are allowed to impinge on a target of chlorine isotope $^{37}_{17}Cl$. After bombardment, the target is found to be radioactive and to emit electrons. The target activity so induced decreases exponentially to one-eighth of its original value in 15 minutes, and after the activity is virtually ceased, sensitive microanalysis capable of detecting elements with atomic numbers in excess of 11 reveals nothing but chlorine in the target.

(i) Determine the half-life of the radioisotope formed by the bombardment. [1 mark ]

(ii) Write down equations which represent the nuclear reactions that could give rise to the observed results.

Note: Elements near chlorine in the periodic table are:

$$_{15}P \;;\; _{16}S \;;\; _{18}Ar \;;\; _{19}K$$

whilst the stable isotopes for chlorine are $^{35}_{17}Cl$ and $^{37}_{17}Cl$. [5 marks]

(b) The heaviest stable isotopes of some typical elements of medium atomic weight are as follows:

$$^{86}_{36}Kr \;;\; ^{93}_{41}Nb \;;\; ^{110}_{46}Pd \;;\; ^{123}_{51}Sb \;;\; ^{138}_{56}Ba$$

(i) Use this information to explain why at least one of the fragments produced by fission of the nucleus $^{236}_{92}U$ must be radioactive.

[4 marks]

(ii) Discuss the nature of the radioactive changes that are initiated from such a radioactive fission fragment.

[4 marks]

**END OF QUESTION PAPER**