NZEST
NEW ZEALAND EDUCATION & SCHOLARSHIP TRUST
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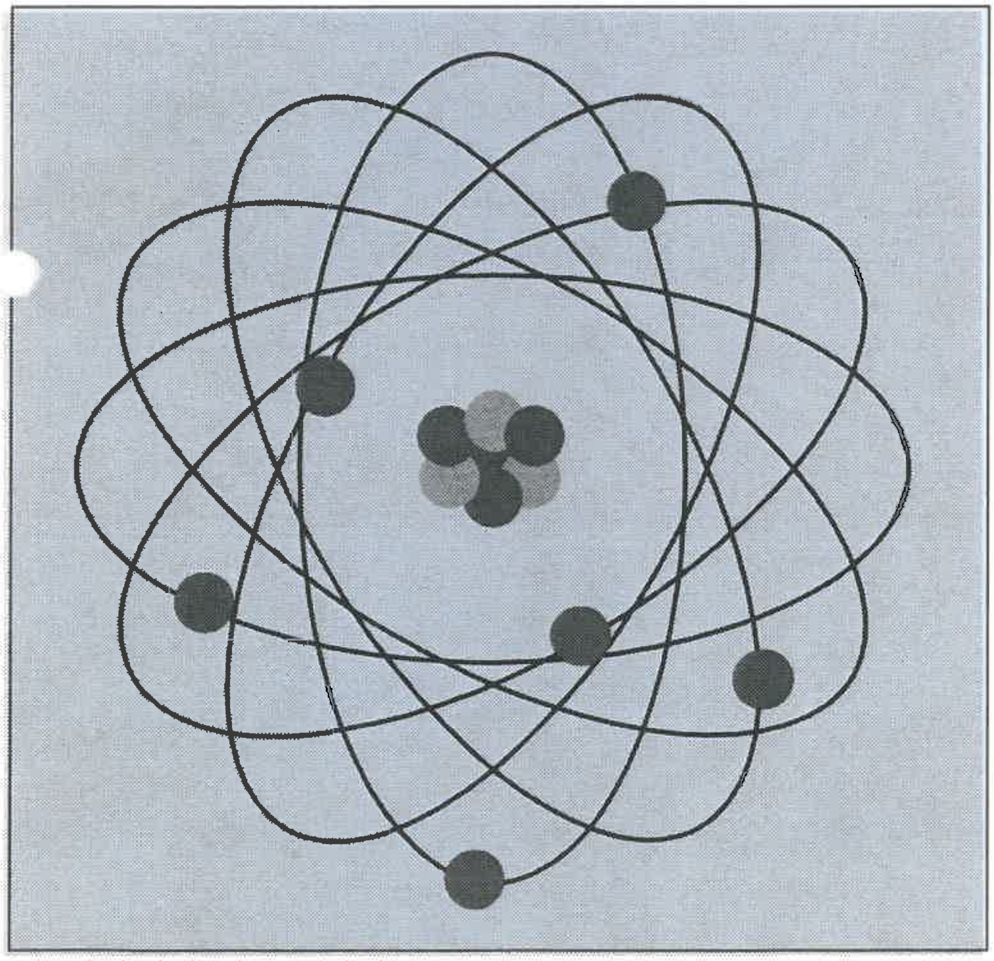
NZEST SCHOLARSHIP EXAMINATION

2000 EXAMINER'S REPORT AND SOLUTIONS

PHYSICS



New Zealand Education and Scholarships Trust

2000 PHYSICS EXAMINER'S REPORT

Contents:

I. Exam Statistics & Comparison with previous years

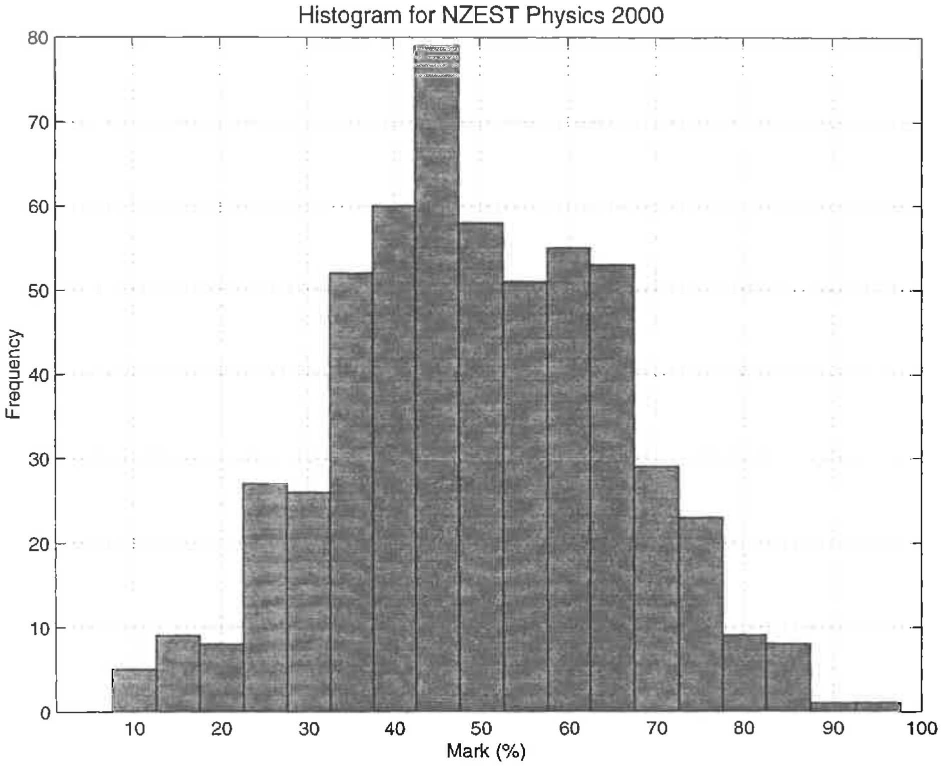
	2000	1999	1998	1997	1996	1995
No. of students	554	642	659	596	595	481
Top score	96%	89%	91%	89%	86%	99%
Upper quartile	69%	57%	61%	54%	53%	80%
Median	**49%**	**45%**	**49%**	**44%**	**42%**	**53%**
Lower quartile	39%	33%	38%	34%	30%	38%

24 students were graded A+ with mark ≥ 77%. The lower bound for a C– grade was set at 35%.

II. Question Statistics for 2000 Paper

	A	B1	B2	B3	B4	B5	B6	B7	B8
Max possible	30	15	15	15	15	15	15	15	15
Top*	31	16	16	14	15	14	16	16	16
Upper qu.	21	11	11	6	10.5	10.5	9	10	9
Median	**18**	**9**	**8.5**	**4**	**7**	**8.5**	**6.5**	**8**	**5.5**
Lower qu.	14	5	7	2.5	4.5	5.5	3	5	3
Mean	17.8	8.3	8.6	4.4	7.2	7.9	6.1	7.6	5.9
Std devn	5.5	3.8	2.8	2.6	3.9	3.1	3.8	3.7	3.9
Bottom	3	0	0	0	0	0	0	0	0
N	554	554	554	554	554	554	554	554	554

* One bonus mark per question could be earned (i.e., one mark available in Section A, and a total of eight marks available in Section B)



III. General Comments from the Examiner

This is my third year as Physics Examiner. I have maintained the same exam format as for 1999, and the same bonus-mark scheme which permits up to 9 discretionary marks (one mark in section A, one mark per question in section B) to be allocated by the marking panel to answers which demonstrate real insight and excellence.

Although the median mark has improved a little to 49% this year (up from 45% in 1999), this is still lower than I'd like. I think that Scott Whineray's 1995 paper, with its median of 53%, hit the right level. This is because the exam serves two prime purposes. First, it provides a national means of ranking the best physics students across New Zealand (as far as this can be measured in a formal exam). Second, it provides candidates with a challenging assessment of their physics knowledge and problem-solving abilities. A hard-working, well-prepared student should expect to receive a mark which is in fair proportion to the effort she or he has expended—hence my wish for a relatively 'high' median score. The job of the Examiner is not just to come up with 'tough' questions, but also to structure those questions in such a way that the competent student can demonstrate what s/he knows.

There were two changes to the physics exam paper this year. Students now work in a combined question/answer booklet (previously candidates wrote in a separate answer booklet) writing their answers in the formatted spaces following each question or part-question. Extra blank pages at the end of the booklet are available for students who need more space than was allocated in the body of the question. This move to a combined booklet seems to have been well received, and certainly made the marking of the scripts much faster and more efficient. I'll be recommending that we retain this innovation for the 2001 exam, and that the fold-out strip at the back of the booklet (for recording student marks) be pre-printed with the question numbers listed in booklet order. (At present the candidate fills out this strip, and the result is often unusable by the marking panel.)

The second change this year was the inclusion, for the first time in the NZEST Physics paper, of the standard list of UB formulas. This was by popular demand and popular vote: a clear 2/3 majority of respondents to the NZEST survey run in early 2000 told us they wanted the formulas included in the paper. In view of the size of this majority, I anticipate that the UB formula list will now be included in the NZEST exam for the foreseeable future.

I conclude this section by quoting a relevant paragraph from my 1999 report which gives some guidance to scholarship candidates:

"The scholarship physics exam is designed to *test general thinking in physics, probing student understanding of basic physics concepts and skills just a little beyond the strict confines of the bursary prescription* (in a horizontal rather than a vertical sense). These "**basic physics concepts**" certainly include material which is part of the form-6 physics syllabus. My advice to candidates is that as part of your preparation for the scholarship physics exam, you should revise not only the form-7 (year-13) syllabus, but also the form-6 (year-12) material you were taught in your form-6 physics class. In addition, you need to be familiar with log/log and semilog graphs to interpret data, know how to do order-of-magnitude estimation, and understand the notion of dimensional consistency of physical equations. (For an example of an equation which is not dimensionally consistent, see the Aldiss version of $E = mc^2$ in question B8 of the 1999 paper.)"

IV. Question-Specific Comments by Marking Panel

This section is a summary of comments made by members of the 19-person marking panel.

A. Guided Short Answer

A1. Motion & force: Reasonable success. Most common responses were circular motion or resistive forces.

A2. SHM: Good success. Most gave acceleration proportional to displacement.

A3. Light-year: Good success. Well laid out explanations.

A4. Some good explanations of destructive interference.

A5. Reasonable success. Most common error was the suggestion that kinetic energy of the electron was proportional to the frequency of the incident light.

A6. Least successful question. Only a few students gave the full correct explanation.

A7. Done well.

A8. Most students got this one wrong! Popular responses were (b) 300 N and (a) 0 N.

A9. Good well laid-out explanations.

A10 The most successful question, though a few gave the correct working then chose (e) deuterium.

- *General comment:s:* The good students gave excellent explanations. About 6 candidates got full marks (30/30), and one superb script earned 31/30. The median mark for this section (60%) was higher than for other parts of the paper.

B1: Units; Momentum; Projectile Motion

(a) Generally well done, though a fair number could not fathom what the examiner wanted here, writing correct material which did not address the question.

(b) A number of students could not distinguish between work and force, and the idea that work requires a vertical displacement of the weights was frequently ignored.

(c) Mostly well done. Some common errors: (i) Taking pole-to-equator distance as 1/2 of circumference; (ii) Writing circumference as πr^2(!)

(d) Many misunderstood this question and explained why a falling ball gains momentum. Many others said that conservation of momentum can only happen in a collision!

(e) (i) Mostly well done, though some simple mathematical errors made.
(ii) Less well done. The *instantaneous* velocity is required here, but many calculated the *average* velocity (net displacement divided by time taken).

(f) Many (often good) students omitted this as they could see that it was a lot of work for one mark. Accelerated frames-of-reference approach was seldom used. A good discriminator.

- *General comment:* A relatively high-scoring question with a 60% median mark.

B2: Rotation and Graphing

(a) Majority had no problem designing a simple test to distinguish the hollow and solid spheres, but were more shaky in quoting general principles to support their thinking.

(b) (i) Well done: vast majority scored 3/3 for the linear velocity graph. One marker felt that the marks allocation was too generous here.

(ii) Determination of angular acceleration and torque was almost universally correct, the only common error being the calculation of rise and run from two points which are close together on the graph (thereby losing accuracy in the calculated gradient). A surprisingly large number of candidates wrote the unit of torque as $kg{\cdot}m^2{\cdot}s^{-2}$ (rather than the expected N·m); one marker was unhappy with the Examiner's ruling that this be marked correct. [*Examiner:* Expressing a compound unit in terms of its SI basis units, while perhaps a little clumsy, is *not* incorrect.]

(iii) Algebraic manipulation of logs was of extremely variable quality—generally more characterised by confusion than clarity. Satisfactorily done by fewer than half of the candidates.

(iv, v) Only about half the candidates could plot a graph on semilog paper correctly. Significantly fewer could determine the gradient, and hardly any could determine the value of k_1. Clearly the use of log paper is not common in schools, which is probably not surprising given the alternatives that are available on calculators and computers. Perhaps exponential-decay (and power-law) problems would be better examined in future by having the candidates plot the log-transformed variables on *linear* graph paper?

B3: Force; Collision; Torque

(a) Many incorrect answers involved the use of impulse. Most found difficulty gaining two marks. The "gentle" pull was better understood than the "sudden" pull.

(b) (i) Students only infrequently referred to momentum conservation *in the absence of external forces*, so lost an 'easy' mark. Kinetic energy non-conservation in inelastic collisions was better understood. (ii) A common mistake was to equate the initial kinetic energy of the bullet to the final potential energy of the block, leading to a bullet speed of 19.8 m/s! [The paper had insufficient space here for student working and diagram.]

(c) (i) Done well; was the easiest part of the question. Some students failed to attach their tension vectors to the ruler. (ii) Few correct answers here. $T_1' = ma_{cm}$ was common. (iii) If completed, usually correct. (iv) The hardest part with a lot of non-attempts. Those who did attempt it usually got through to an expression for α.

- *General comment:* Despite the fact that parts (a) and (b) tested understanding of F6 mechanics, students did not score well. Overall, students found this to be a relatively hard question, and it was done poorly (median 27%).

B4: Tides and SHM: Gravity

(a) (i) Generally well done. Some students did not realize the amplitude was 2 m (wrote it as 4 m). Others had trouble determining the phase constant.

(ii) Students who got incorrect values for ω and ϕ had difficulties simplifying the trig formula.

(iii) About half the students obtained the correct times for maximum rate of tide height, but only a few realized that $v_{max} = \omega A$ was the appropriate equation to use.

(iv) Simple trigonometry, but many students could not do it. Only a third drew a diagram, and only a few of these had $\pi/3$ m/h as the vertical element of the velocity right-triangle.

(v) The few students who drew a reference circle generally got sensible answers here. Many used the displacement-time equation, but then failed to convert from number of hours to time-of-day (and so lost a half-mark).

(b) Many students forgot to square the radius, or used incorrect values for r_A and r_B (e.g., 3999 and 4001). Some rounded too early, giving Δg as zero, consequently missing the point of the question. However, there were some very impressive answers presented in this part.

B5: Electric Circuits

(a) (i) Answered correctly by vast majority of students. Commonest error was reading V_{max} as 12 volts.
(ii) Again well answered. Common error: incorrect conversion from ms to s. Most knew $f = 1/T$ and used it correctly.

(b) Part (i) was well answered, but in part (ii) not one student realized that the rectified waveform would appear inverted. Many incorrect answers showed a capacitor-smoothed wave.

(c) Well answered by most. Students tended either to get the full 3 marks or none. This is a good question, but could have had fewer marks allocated to it.

(d) Many students cannot enter 10^{20} into their calculator correctly*! Others failed to multiply by the electron charge.

(e) (i) Many candidates knew that 'earthing' was dangerous, but there was some confusion with 'earthed appliances'. Few commented on the danger to the heart. (ii) Many thought that the technician *had* to be earthed. (iii) Not many recognized that the potential difference across the legs of the bird is very small.

(f) If only a value for the body resistance had been given in question! Then the question could have proved far more discriminating. Many students could do the initial calculation to get a capacitor voltage of 2390 V, but rather few recognized the need for a short *RC* time-constant in order to deliver the required energy in 2 ms.

* **Teaching point with calculators:** Make sure students understand how to enter 'pure' powers-of-ten on the calculator. For example, 10^6 is *not* entered as:

[1] [0] [Exp] [6]

B6: AC; Magnetic Field; Reactance

(a) The marking schedule was altered to give 1 mark for part (ii) and 3 marks for part (iii). This was done to accommodate student unawareness of n being turns per metre, since the given formula ($B = \mu_0 nI$) is less familiar than $B = \mu_0 NI / \ell$. Students who gave a good explanation and a diagram for part (iv) received a bonus mark. Part (v) was marked liberally so that students who misread $F = kI_1 I_2 \ell / d$ as $F = kI_1 I_2 1/d$ were not penalized.

(b) Some students showed a lack of understanding of impedence and vector manipulation by claiming that if $X_C = 50\ \Omega$ and $R = 50\ \Omega$ then $Z = 100\ \Omega$ (!)

- *Marker comment*: Many students did not attempt this question, and those that did either scored well or poorly. This could indicate a lack of confidence with electromagnetism in many candidates?

B7: Gravity; Doppler; Interference

(a) (i) Majority of students did not know that the orbit must include the centre of Earth. Many thought that because Russia is in the northern hemisphere that only orbit III would be possible (this assumes a north-south orientation on the diagram which is not actually shown; also, in principle, the Russians *could* have launched from an equatorial site).

(ii) Well done, except that almost all gave the orbit height to 3 sig. figs, so losing a half-mark. (When subtracting two nearly equal numbers, there will generally be a loss of significant digits. Here, for example, (6.65 – 6.37) = 0.28, a 2-sig. fig. result from 3-sig. fig. data.)

(iii) Most students were good on this.

(iv) Quite well done—only a few over- or under-estimated the graph plateaux.

(v) Surprisingly well done for a difficult calculation. Very few realized that the low calculated satellite speed arises because the satellite's velocity vector is not directly towards the receiver; this last part was a good discriminator.

(b) Much confusion over use of formulae and correct units. Many obtained 4250 Hz but failed to gain the last mark for the higher harmonics heard. Another good discriminator.

- *Marker comment*: A good question with plenty of challenge for the above average student. Only three or four obtained the discretionary bonus mark. This points to a general lack of knowledge about satellite orbits and associated theory.

B8: Photons; Decay; Estimation

(a, b) Generally done well.

(c) Many students could not cope with the logarithm manipulations required to date the wood sample.

(d) (i) Very few could relate the speed of the meteor to the gravitational attraction of Earth or to the loss of gravitational PE falling in from infinity. (Perhaps not surprising since it is not taught in bursary.) (ii) Students were not very good at giving a plausible guess for the density of the meteor. Some came up with a density which is lower than that for air at standard conditions! A number did not know the formula for the volume of a sphere.

- *Marker comment*: This question scored poorly (median 37%). Perhaps candidates were running out of time by this point of the paper?

V. General Comments from the Marking Panel

- In the back flap of the answer book, please **pre-print the question numbers** "A B1 B2... B8" to avoid confusion when recording marks. [**Examiner:** The same request was made last year, but was overlooked, sorry.]
- The new format of combined question/answer Booklet made marking *so* much easier and faster—thank you. [Several members of the marking panel made similar comments.]
- For the 2001 paper, allow more space for working and diagrams following each question.
- Candidates need to be reminded that there are empty pages at the end of the Booklet if they run out of space [see Instruction 4 on the front of the Booklet]. Those who had tied in extra pages had left the spare pages blank.
- Thanks for the list of bursary formulae—but NZEST administration should have ensured that all schools knew of this change. [**Examiner:** All schools were surveyed early in 2000 to seek guidance as to whether or not the formulae should be included. A clear majority of respondents said yes.]
- Thanks again for the comfortable chairs—I'm convinced that perching on lab stools slows the work rate!
- Congratulations on an interesting, well-presented and realistic exam paper.
- Thanks for writing another excellent exam! I enjoyed working through it although I didn't gain a mark of 144/150... [This was the mark achieved by the top scholar.]

VI. Survey Responses

Several senior physics teachers were surveyed for their appraisal of the exam. Here is a summary of their responses.

1. Was the Scholarship examination at the right level, i.e., sufficiently challenging, to extend the able student?

Three respondents said "**yes**" or "**about right**". Other responses:

- There appeared to be a range of difficulty allowing some discrimination.
- Most definitely at the right level and well within the scope of a candidate expecting to score 75+ in the Bursary exam.
- Yes. Perhaps slightly easier than that set over the last 2 or 3 years, but still challenging and within the scope of a good bursary student.
- Yes. Seemed slightly easier than the 1999 exam; nevertheless an interesting set of questions.
- Yes. Good level of challenge in terms of both physics concepts and mathematical processes.
- Yes. It was approachable for all the well-prepared students, and had that necessary lateral twist throughout.
- Yes, but only just [sufficiently challenging]. No further easing of the level.

2. Were there any questions considered to be inappropriate?

Three respondents gave an unqualified "**no**". Other responses—

- No. As a scholarship exam for the top students I thought it was great.
- No. All were within the notes supplied earlier in the year (e.g., semilog graphing).
- B6 (a) (iii and v) are not in the curriculum, and n was not defined in (iii). Examiners should be able to work within the curriculum and find sufficient challenge there.
- B6 (a) (iii): The formula $B = \mu_0 nI$ needed clarification as its derivation is no longer in the syllabus. The force formula in part (v) suffered from a font-change problem in which the 'ℓ' became an '1'. [**Examiner:** My apologies for not picking up this printery error.]
- Form 6 concepts like those in projectiles [B1(e),(f)] and force between current-carrying parallel wires [B6(a) (iii),(iv)] have been included. This means that students are expected (as prescribed by the Scholarship syllabus) to revise Year-12 concepts. [**Examiner:** Yes, this is a deliberate inclusion.]
- Students are expected to have a knowledge of semilog graph papers, which is not expected in the Bursary exam. Is this a 'must' to extend the able students? [**Examiner:** That is the intention.]
- A6: Option (e) [that 'magnetic fields do not exist'] is too vague. [**Examiner:** But option (e) is a distractor, so it is permissible for this bit to be humorously vague, I think.]
- B7 (a) (iii, iv, v): Except for the end of part (v), these test maths ability, not physics. [**Examiner:** But not unreasonably, in my view. Part (iii) is a two-line ratio argument; part (iv) asks students to read values from a graph. There are 3 marks in part (v) for a detailed calculation which the best students did well.]

3. Was the coverage of the syllabus satisfactory?

Four respondents gave an unqualified "**yes**". Other responses—

- Overall very good
- Excellent, though perhaps atomic and nuclear could have had a little more exposure
- Okay, but modern physics and waves were not covered well
- AC theory almost absent [**Examiner:** What about B5(a), (b) and B6(b)?]

- Mechanics was overemphasized. The concentration on Year-12 content was excessive even taking account of the examiner's declared prediliction for this mixture.
- There were no questions on electromagnetic induction, Bohr's theory, capacitances in series and parallel, standing waves, angular momentum, rotational equation of motion [**Examiner:** Regarding rotational motion, what about B3(c)?]

4. **Length of the paper — was the exam perceived to be too long? too short? just right?**

Six respondents said "**just right**". Other responses—

- Possibly too long for the average student
- If you know what you are doing it's all easy, I suppose!
- This paper appears to take 5 to 6 hours to complete. Students who can complete the whole paper in 3 hours must be really sparkling brilliant.

5. **What could be done to improve this Scholarship exam paper?**

- Focus better on the Year-13 core syllabus.
- Do not contradict UB requirements: you ask for 3 sig. figs unless stated otherwise, then use only 1 sig. fig. in several questions (e.g., B4(b)).
- Give the students a little more 'doodle' room!
- Ask more conceptual questions, fewer complicated maths-type questions.
- Do *not* issue list of formulae—students need to learn these, and be able to derive key results. [**Examiner:** I agree. However, the school survey in early 2000 indicated that a very significant majority of physics teachers wanted the formula list to be included, so I reluctantly made the change.]
- Don't use a combined question/answer booklet. [**Examiner:** The change to the combination booklet was made at the request of NZEST to ensure a consistent look-and-feel across the range of NZEST exam papers. A welcome side-effect was that the resulting scripts were very much easier to mark than with the previous generic booklet. The marking panel liked working with the combined format, so I will be **recommending that the combination booklet be *retained* for 2001**.]
- Use standard symbols for circuit components (e.g., show resistors as a rectangle rather than as a zig-zag).
- Give more diagrams and clarifications (e.g., ballistic pendulum of B3(b) needed a picture).
- Make the exam harder and more challenging! [**Examiner:** I'd urge caution here. For the 2000 exam, the median came out at 49%, which is *lower* than I'd like. The purpose of the exam is two-fold: First, to produce a national ranked list, and second, to reward toiling students with a sense of satisfaction for honest hard work in what is already a very challenging physics exam.]

General comments?

- A very good exam. Keep it up.
- A good challenging exam for the knowledgeable and *very* quick-witted.

- I was not aware that formulae would be given, nor that the format would change to a combined question/answer booklet. These changes are positive, but I need to be able to tell the students before the exam.
- I enjoyed the paper and its style of short-answer questions testing a wide range of physical concepts.
- I liked the answer-on-the-paper format.

VII. Acknowledgements

My sincere thanks to my moderators and to the marking panel of senior physics teachers who gave me several ideas for nice scholarship questions. Also thank you to Sarah Taylor, Executive Officer NZEST for her friendly and efficient management. It has been a pleasure working with colleagues who both care about physics and seek to encourage excellence in their students.

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30 December, 2000

NZEST Scholarship Physics 2000

Model Answers and Marking Scheme

Total marks available: 150

Section A: Guided Short-Answer [3 marks each]

Two marks for reasoning; one mark for correct T/F or multichoice selection.

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[3] 1. **False** For example, for uniform circular motion, force is radially inwards while velocity vector is tangential. Other examples: Rising projectile [motion is upwards but net force is downwards]; any decelerated motion [force acts to reduce velocity].

[3] 2. **False** Acceleration is *not* constant for SHM. In fact, $a \propto -x$. Therefore kinematic equations of motion cannot be used here.

[3] 3. **True** Distance travelled in 1 ns $= ct = (3\times10^8)(1\times10^{-9}) = 0.30$ m.

[3] 4. **True** Provided the sound sources are *coherent* (i.e., same frequency, fixed phase difference between them), then for a point in space for which the two sources produce disturbances which are equal in amplitude and opposite in phase, a silence (cancellation by superposition) will occur.

[3] 5. **False** Intensity (*number* of photoelectrons per second) provides no information on *energy* of the incident light. Kinetic energy of the photoelectrons is the surplus of the photon energy over the work required to liberate the electron from the crystal lattice: $E_K = hf - \phi$.

[3] 6. **d** By the right-hand slap rule, a magnetic field can change the *direction* of the charge's velocity, but cannot change its *speed* and therefore cannot change its KE. This means that a magnetic field cannot do work on a charge.

[3] 7. **e** Given the second and third facts, the time for the Moon to rotate once on its axis must match the time to orbit Earth. [The first fact is a distraction!]

[3] 8. **d** The balance records the tension in the right-hand string, $T = mg \approx 150$ N. [One could replace the left-hand string and mass with a wall or other fixture without altering the balance reading.]

[3] 9. **a** For circular motion, centripetal acceleration is $a = v^2/g$. Setting $a = g$ and solving for speed gives $v = \sqrt{gr} = \sqrt{150} = 12.2$ m/s $= 44.1$ km/h.

[3] 10. **b** Balancing nucleon and charge numbers gives ${}^4_2$X which identifies a helium nucleus ${}^4_2$He, also known as an alpha-particle.

Section B: Long Questions [15 marks each]

One bonus mark per long question available if student answer shows exceptional insight. Mark subtotals are shown boxed in left margin.

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1. **Units; Momentum; Projectile Motion**

[1] (a) Yes. 1 N corresponds to a mass of about 100 g which is probably fairly typical for an average apple.

[2] (b) No. Assuming the weights are not moving vertically, then there is no displacement in the direction of the applied force, so no work is done on the weights. [The muscles tire because of the contractions/relaxations required to maintain the stressed muscular state, but none of this 'microwork' is transferred to the weights.]

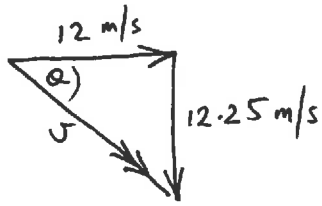
[2] (c) The distance from North Pole to equator is one-quarter of the Earth's circumference:

$$\frac{1}{4} \times \text{circumference} = 10^7 \text{ m}$$
$$\text{so} \quad 2\pi r = 4\times 10^7$$
$$\therefore \quad r = \frac{4\times 10^7}{2\pi} = 6.37\times 10^6 \text{ m.}$$

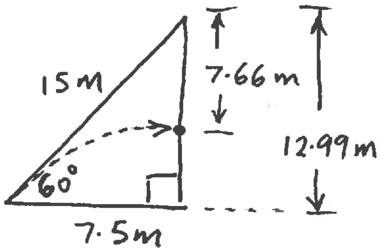
[2] (d) Momentum is conserved if there is no net external force acting on the system (i.e., the system is *isolated*). In this case, gravity is an external force acting on the ball. [If the system were enlarged to include Earth, then the total momentum of the Earth–ball system *would* be conserved since the gravitational interactions (Earth attracts ball; ball attracts Earth) would constitute *internal* forces, so momentum changes would sum to zero.]

[4] (e) i. Time to travel 15 m horizontally is given by $t = x/v_x = 15/12 = 1.25$ s. Distance fallen in this time is $y = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2}(9.8)(1.25)^2 = 7.66$ m. Both dart and monkey begin falling from rest at the same instant with the same acceleration ($= g$), so maintain a common altitude. Therefore the dart must hit the monkey. [This assumes that the initial height of the monkey is $h \geq 7.66$ m; if $h < 7.66$ m, then the dart will hit the ground before covering the horizontal 15 m to reach the monkey.]

[3] ii. Vertical speed after 1.25 s is $v_y = gt = (9.8)(1.25) = 12.25$ m/s. Applying Pythagorus to the velocity triangle, $v^2 = v_x^2 + v_y^2 = 12^2 + 12.25^2$, so $v = 17.1$ m/s, at angle $\theta = \tan^{-1}(\frac{12.25}{12}) = 45.6°$ below the horizontal.



[1] (f) The horizontal component of dart velocity is $v_x = 12\cos 60° = 6$ m/s. The horizontal distance to the line-of-fall of the monkey is $x = 15\cos 60° = 7.5$ m. The time to travel this distance is $t = 7.5/6 = 1.25$ s. In this time, the monkey falls 7.66 m from initial height of 15 sin 60° = 12.99 m. So the dart hits the monkey at an altitude of $12.99 - 7.66 = 5.33$ m.



OR

In time t, dart falls below its 'zero-gravity' trajectory by amount $\Delta y = \frac{1}{2}gt^2$, exactly matching the vertical fall of the monkey. Thus, in the frame of reference of the monkey, the dart moves directly towards the monkey, so the dart must collide with the monkey.

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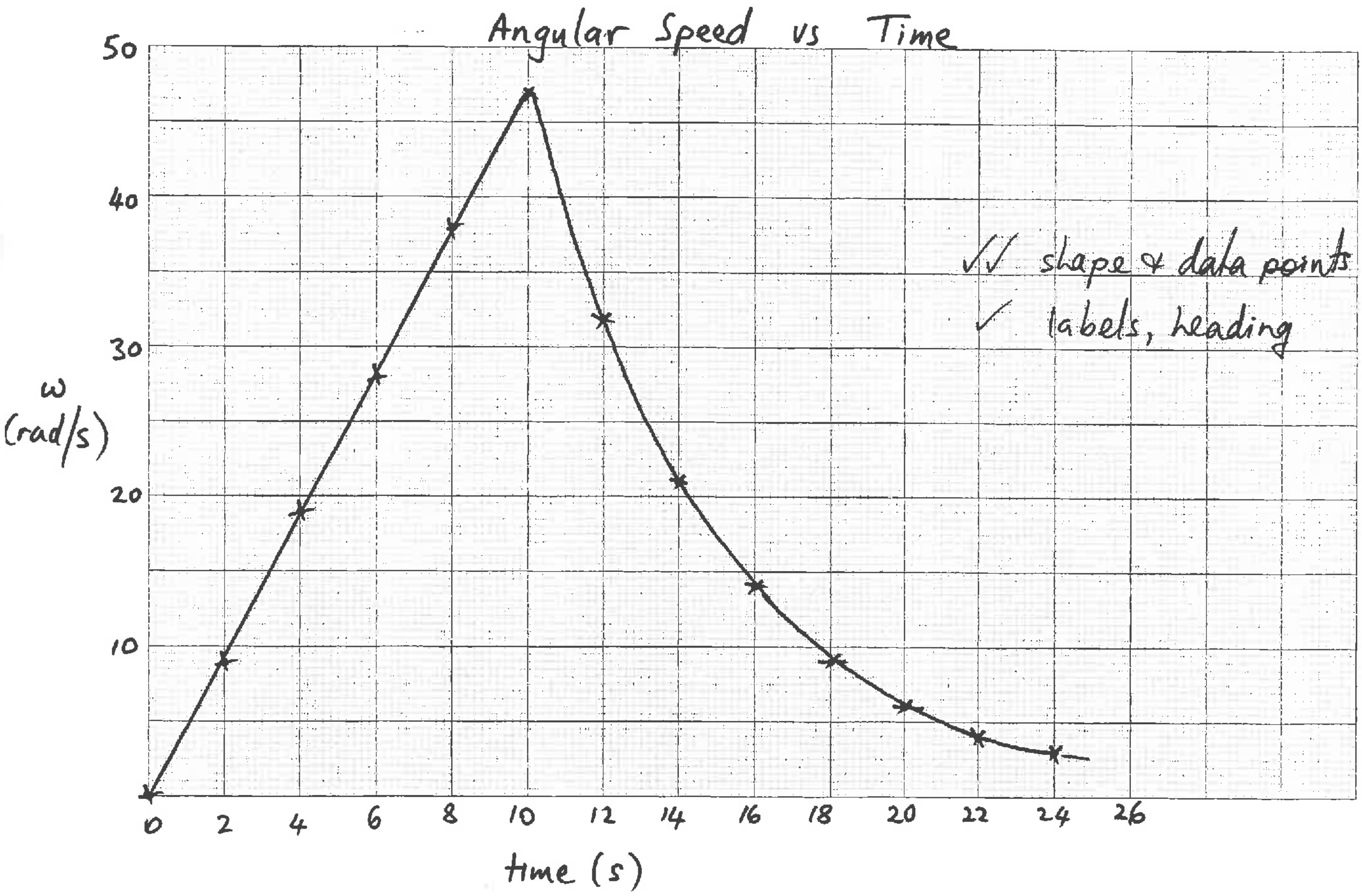
2. Rotation and Graphing

[3] (a) **Principle:** The hollow sphere has its mass distributed further from its centre, so has the larger rotational inertia and will accelerate more slowly when rolling down an inclined plane. **Method and prediction:** Roll both spheres a measured distance down an inclined plane. Record the travel times. The hollow sphere will have the slower time.

[3] (b) i. **Graph of ω–vs–t on linear paper**

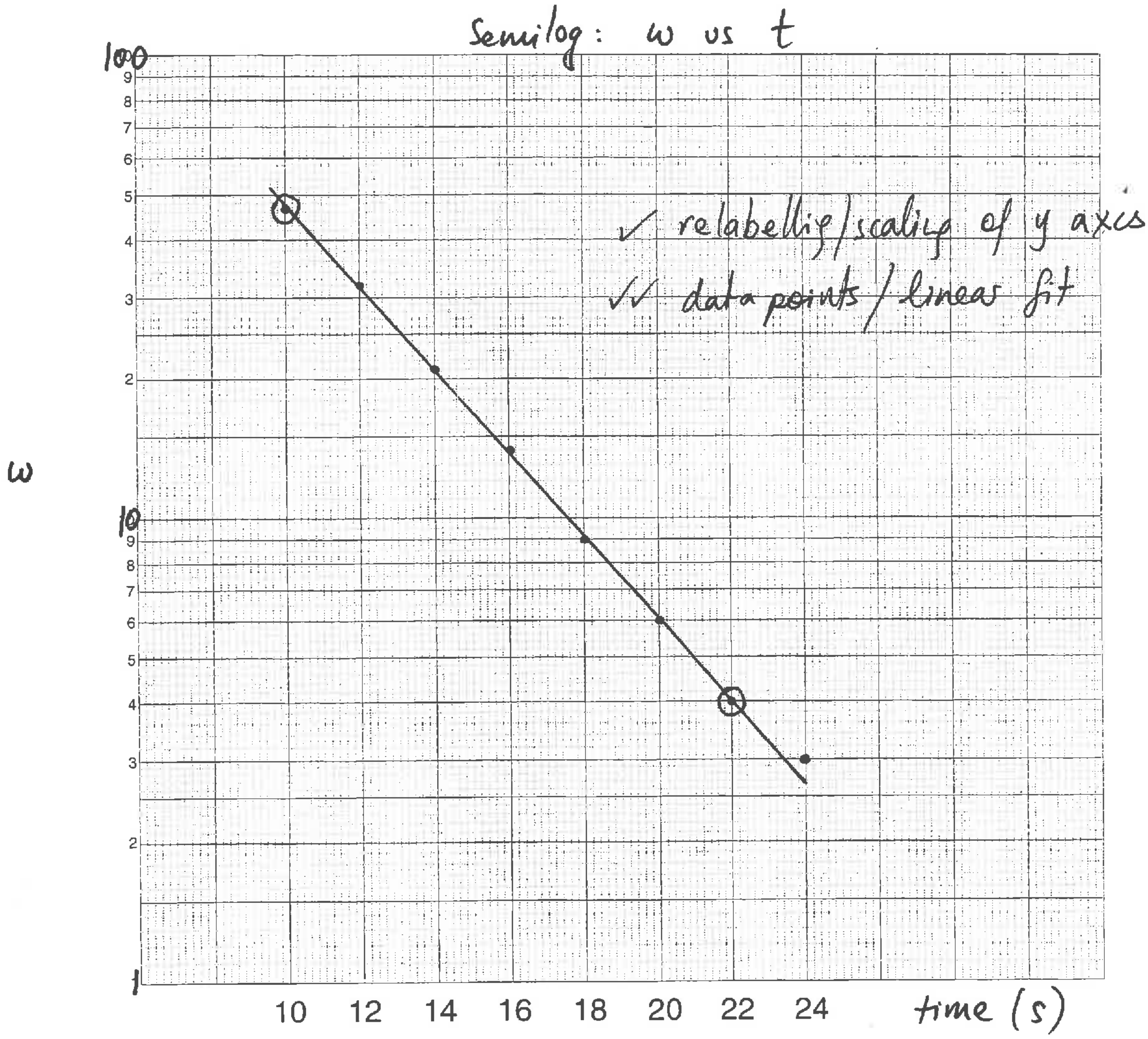


2 ii. The angular acceleration is given by the gradient of the graph: $\alpha = \Delta\omega/\Delta t = (47-0)/(10.0-0.0) = 4.7\,\text{rad/s}^2$. The required torque is:

$$\tau = I\,\alpha = (4.0\times10^{-2})(4.7) = 0.188\,\text{Nm}.$$

2 iii. Taking natural logarithms of both sides of the decay equation $\omega = k_1\,e^{k_2(t-10)}$ gives $\ln\omega = \ln k_1 + k_2(t-10) = k_2 t + (\ln k_1 - 10k_2)$. This corresponds to a straight line $Y = mX + c$ in which the slope is $m \equiv k_2$ [and $c \equiv \ln k_1 - 10k_2$ gives the axis intercept at time $t - 10 = 0$, i.e., at the instant $t = 10$ when the exponential decay began].

3 iv. **Graph of ω–vs–t on semilog paper**



2 v. To find k_1, set $t = 10$ in the given decay equation, giving $\ln\omega = \ln k_1$, so $k_1 = \omega|_{t=10} = 47\,\text{rad/s}$. The slope is $\Delta Y/\Delta X$ where $Y \equiv \ln\omega$ and $X \equiv t$, so

$$\text{slope} = \frac{\Delta\ln\omega}{\Delta t} = \frac{\ln\omega_2 - \ln\omega_2}{t_2 - t_1} = \frac{\ln 4.0 - \ln 47}{22-10} \quad \text{(using circled points)}$$

$\therefore\quad k_2 = -0.205\,\text{s}^{-1}$.

3. **Force; Collision; Torque**

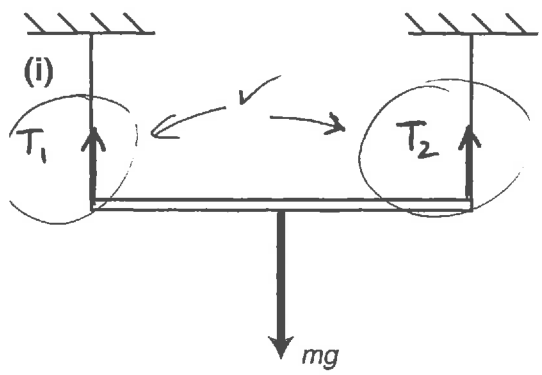
[2] (a) **Block on a string** For a gentle pull, tension in the upper string is larger because of the weight of the block, so the upper string breaks first. For a sudden pull, inertia of the block resists acceleration, so tension in the lower string increases to breaking point before block has moved sufficiently to stretch the upper string.

(b) **Ballistic pendulum**

[2] i. Kinetic energy is *not* conserved because friction between block and bullet transforms most of the KE to heat (inelastic collision). Momentum *is* conserved because, during the very brief duration of the collision, the external forces (string tension, gravity) have negligible effect.

[2] ii. Let V be the speed of the bullet/block combination immediately after the collision. The kinetic energy immediately after collision is completely transformed to gravitational potential energy at the top of the arc, so $\frac{1}{2}(m+M)V^2 = (m+M)gh$. Solving for V gives $V = \sqrt{2gh} = 0.990\,\text{m/s}$. Applying momentum conservation to the collision itself, the bullet momentum mv prior to collision becomes bullet/block momentum $(m+M)V$ immediately after collision: $mv = (m+M)V$, so $v = \frac{m+M}{m}V = 397\,\text{m/s}$.

(c) **Suspended metre ruler**



[2] i. Tension in left-hand (and right-hand) string is $T_1 = mg/2$.

[1] ii. Force equation relating new tension T_1' and CM acceleration a_{CM} is $mg - T_1' = ma_{\text{CM}}$.

[2] iii. Relationship between angular acceleration α and CM acceleration: $a_{\text{CM}} = r\alpha = 0.5\alpha$ (since CM is 0.5 m from left-hand end).

[4] iv. Taking moments about the left-hand end gives a torque $\tau = mg \cdot L/2$ where $\tau = I\alpha$, and $I = \frac{1}{3}mL^2$ is the moment of inertia of a rod about one end (refer second diagram at bottom of page 11 of the question paper). Setting $L = 1.0\,\text{m}$ and solving for angular acceleration gives $\alpha = \frac{3}{2}g$. Using the result from part (iii) gives linear acceleration $a_{\text{CM}} = 0.5\alpha = \frac{3}{4}g$. Hence, from part (ii), the altered tension is $T_1' = mg - \frac{3}{4}mg = \frac{1}{4}mg$.

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4. **Tides and SHM; Gravity**

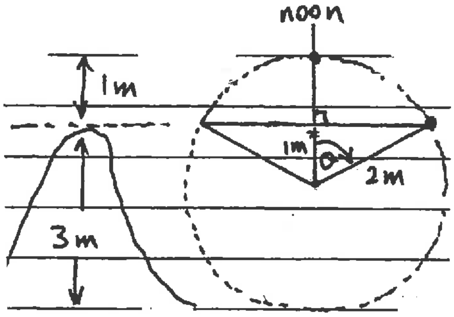
[3] (a) i. The SHM is of amplitude $A = 2\,\text{m}$. [Note that $y(t) = A\sin(\omega t + \phi)$ gives the SHM variation in height *about the 2.0-m mean* level, thus the absolute height is given by $h(t) = 2.0 + y(t)$.] The period of the SHM is $T = 12\,\text{h}$ so the angular frequency is $\omega = 2\pi/T = 2\pi/12 = \pi/6\,\text{rad/h}$. The phase constant is $\phi = \pi/2\,\text{rad}$ (this shifts the sinewave a quarter-cycle to the left, giving a cosine wave).

1 ii. Substituting into the given trig identity gives,

$$\begin{aligned} y(t) &= A\,(\sin\omega t\,\cos\phi + \cos\omega t\,\sin\phi) \\ &= A\,(\sin\omega t\,\times 0 + \cos\omega t\,\times 1) \\ &= A\cos\omega t\,. \end{aligned}$$

3 iii. The maximum rate of change in tide height occurs at half-tide, i.e., at 3 pm, 9 pm, 3 am, 9 am. This maximum value is given by $v_{\max} = A\omega = (2)(\frac{\pi}{6}) = 1.05$ m/h $= 0.029$ cm/s.

1 iv. In 1 s, height increases by 0.029 cm, so distance advanced along the slant path is given by $\sin 5° = 0.029/\ell$, so $\ell = 0.029/\sin 5° = 0.33$ cm. Thus the speed of advance is 0.33 cm/s (or 12.0 m/h).



3 v. From the reference circle, the boulder is exposed when $\cos\theta = \frac{1}{2}$, i.e., when $\theta = 60° = \frac{1}{6}$ rev, corresponding to a time interval $\Delta t = T/6 = 2$ h. Therefore the boulder first becomes visible at 2 pm.

4 (b) **Black-hole gravity**

Working in distance units of metres, let $r = 4\times 10^3$ be the distance from the centre of the ruler to the centre of the black hole. Then we can write the distance from the black hole to the nearer end of the ruler as $r_A = r - 0.5$, and to the other end of the ruler as $r_B = r + 0.5$. The difference in gravitational field strength between the two ruler ends is given by

$$\begin{aligned} \Delta g &= GM\left(\frac{1}{r_A^2} - \frac{1}{r_B^2}\right) = GM\left(\frac{r_B^2 - r_A^2}{r_A^2\,r_B^2}\right) = GM\frac{(r_B - r_A)(r_B + r_A)}{r_A^2\,r_B^2} \\ &= GM\frac{(1)(2r)}{(r-0.5)^2(r+0.5)^2} \approx \frac{2GMr}{r^4} = \frac{2GM}{r^3}. \end{aligned}$$

Substituting numerical values for the constants gives

$$\Delta g = \frac{(2)(7\times 10^{-11})(2\times 10^{30})}{(4\times 10^3)^3} = \frac{7}{16}\times 10^{10} = 4.4\times 10^9 \text{ N/kg}.$$

This is a *huge* gravitational gradient. The ruler will be progressively stretched ("spaghettified!") until it breaks up well before it reaches the 3-km event horizon.

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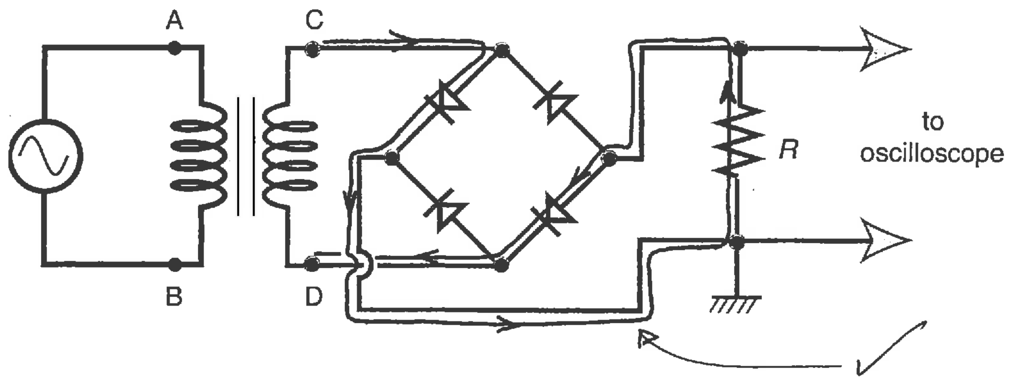
5. **Electric Circuits**

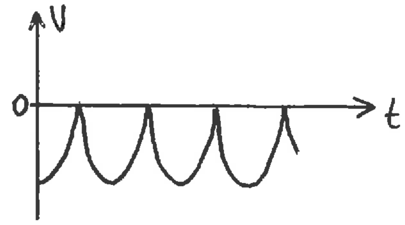
1 (a) i. The rms voltage is $V_{\rm rms} = V_{\max}/\sqrt{2} = 6/\sqrt{2} = 4.24$ V.

1 ii. From the oscilloscope trace, two cycles are completed in 0.5 ms, i.e., $2T = 0.5$ ms, so $T = 0.25$ ms, giving frequency $f = 1/T = 4000$ Hz.

1 (b) i. **Conventional current in rectifier circuit**



1 ii. Note that the direction of the conventional current is *from* the earthed end of the resistor *towards* the unearthed end monitored at the oscilloscope, therefore the upper end of the resistor must be at the *lower* potential, so the trace will be an upside-down rectified sinewave.



3 (c) The two resistors have values 4.0 Ω and 12.0 Ω. To produce the other two required values, placing the resistors in parallel gives $[4^{-1} + 12^{-1}]^{-1} = 3.0\,\Omega$, and placing them in series gives $4 + 12 = 16\,\Omega$.

2 (d) The average current is total charge flow per time taken, $I = \Delta Q/\Delta t$. The charge per electron is 1.6×10^{-19} C, so the total charge delivered in the strike is $\Delta Q = (1.6\times10^{-19})(1\times10^{20}) = 16$ C, giving an average current of $\Delta Q/\Delta t = 16/0.02 = 800$ A.

1 (e) i. Any current path across the chest (e.g., along an arm, through the cardiac region to either the opposite arm or through legs to Earth) since heart function may be disrupted.

1 ii. The technicians are insulated from electrical "ground" by suitable clothing and/or support systems (e.g., trolleys running along the high-voltage wires, or suspended from helicopters).

1 iii. Although the bird's body is at high potential, there is negligible potential difference across the legs of the bird, so essentially no current through the body of the bird.

3 (f) Assume that all of the energy from the capacitor is delivered to the patient, so set the capacitor energy to 200 J: $\frac{1}{2}CV^2 = 200$, so $V = \sqrt{(2)(200)/70\times10^{-6}} = 2390$ V. But if we take the resistance of the body as $R_{\text{body}} \approx 5000\,\Omega$, then the RC time-constant is $\tau = RC = (5\times10^3)(70\times10^{-6}) = 350$ ms, so it is not possible for all of the 200 J to be delivered in 2 ms. This suggests that the capacitor will need to be charged to considerably more than 2.4 kV.

6. **AC; Magnetic Field; Reactance**

[2] (a) i. Expected current draw is $I = V/R = 230/38.0 = 6.05$ A. The corresponding power consumption would then be $P = V^2/R = (230)^2/38.0 = 1392$ W(!)

[2] ii. The predicted power consumption is vastly greater than the 100-W manufacturer's rating, and far outside the 5% error tolerance. In fact, the calculation in part (i) is incorrect because the resistance of a bulb filament is not constant: it increases strongly with temperature. Using $R = V^2/P$ with $P = 100$ W gives an operating resistance of 529 Ω (a factor of ~14 increase over the measured room-temperature resistance).

[2] iii. In the solenoid equation $B = \mu_0 n I$, the value n is the turns density (number of turns per unit length). Here, $n = 3$ turns/mm $= 3000\,\text{m}^{-1}$. The magnetic field strength is then $B = (4\pi\times10^{-7})(3\times10^3)(5.0) = 1.88\times10^{-2}$ T.

[2] iv. Adjacent turns have parallel currents travelling in the same direction. The interaction force will be attractive since "like currents attract". [Can show this be considering the force of the magnetic field from wire-1 interacting with the charges flowing in wire-2, and vice versa.]

[2] v. The length of a single turn is $\ell = 2\pi r = 2\pi\frac{1}{2}(0.5\times10^{-3}) = 1.57\times10^{-3}$ m. The distance between adjacent turns, $d = \frac{1}{3}$ mm $= 3.33\times10^{-4}$ m. Applying the given force equation, $F = kI_1I_2\ell/d = (2\times10^{-7})(5)(5)(1.57\times10^{-3})/(3.33\times10^{-4})$ $= 2.36\times10^{-5}$ N.

Note: Due to a font-substitution error at the printery, the "ℓ" in the given equation appeared in the paper as "1" which many students took as the numeral one (1). Students were not penalized for this.

[2] vi. Note that the magnetic interaction force is proportional to the square of the current, $F \propto I^2$, so, because the startup current is ~14 times larger than the steady-state current, the interaction forces at startup will be $14^2 \approx 200$ times larger than that calculated in part (v), leading to excessive flexing and subsequent mechanical failure.

OR

Maximum rate of expansion of the filament occurs as it heats up from room temperature, so the warm-up phase will be the time of maximum thermal and mechanical stress.

[3] (b) To calculate the current drawn, we need to know the total impedence $Z = \sqrt{X_C^2 + R^2}$ of the circuit. The capacitive reactance is $X_C = 1/\omega C = 1/[(2\pi 50)(63.7\times10^{-6})] = 50.0\,\Omega$. i.e., at 50 Hz, the capacitive and resistive components have equal impedence, so $Z = \sqrt{50^2 + 50^2} = 50\sqrt{2} = 70.7\,\Omega$. Hence the rms current is given by $I_{\text{rms}} = \varepsilon_{\text{rms}}/Z = 230/70.7 = 3.25$ A(rms).

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7. **Gravity; Doppler; Interference**

[3] (a) i. Only II and III are possible since the centre of the Earth must lie within the orbit plane of the satellite.

[2] ii. For a circular orbit, orbit speed is $v =$ (circumference)/(period) $= 2\pi r/T$ where $T = (90)(60) = 5400$ s, so $r = vT/(2\pi) = (7740)(5400)/(2\pi) = 6.65\times10^6$ m. Subtracting the Earth radius gives the height of orbit above Earth's surface, altitude $= (6.65 - 6.37)\times10^6 = 0.28\times10^6$ m $= 280$ km.

[1] iii. The larger Doppler frequency f_2 will be associated with the smaller denominator, and vice versa for the smaller frequency f_1, so

$$\begin{aligned} f_2 &= cf/(c-v) \\ f_1 &= cf/(c+v)\,. \end{aligned}$$

Taking the ratio of this pair of equations gives

$$\frac{f_2}{f_1} = \frac{cf}{c-v} \cdot \frac{c+v}{cf} = \frac{c+v}{c-v}\,.$$

[1] iv. Reading from the graph gives Doppler frequencies

$$\begin{aligned} f_1 &= 20,004,560\ \text{Hz} \\ f_2 &= 20,005,520\ \text{Hz}. \end{aligned}$$

[4] v. To simplify the algebra, let $k = f_2/f_1$ in the equation given in part (iii); thus $k = (c+v)/(c-v)$. Solve this equation for v to give

$$\begin{aligned} v &= \frac{k-1}{k+1}c \qquad \text{where } k = f_2/f_1 = 1.000047989 \\ &= \frac{(4.7989\times10^{-5})(3\times10^{8})}{2.000047989} = 7198\ \text{m/s} \approx 7200\ \text{m/s (to 2 sig. figs.)} \end{aligned}$$

This speed is a little lower than that quoted in part (ii) because the Doppler effect only applies to that component of the velocity which is aimed *directly towards* the Earth receiving antenna, i.e., the method detects only the *radial* component of velocity. (The *tangential* velocity component does not produce a Doppler effect.)

[4] (b) Interference maxima will occur when $d\sin\theta = n\lambda$ (i.e., when the path difference from adjacent sources is a whole number of wavelengths) where $\sin\theta = \sin 30° = \frac{1}{2}$, $\lambda = c/f = 340/f$, and the slit separation is $d = 0.16$ m. Substituting these values into the interference equation and solving for frequency gives $f = n(2)(340)/0.16$. Setting $n = 1$ gives the lowest frequency for an intensity peak: $f_1 = 680/0.16 = 4250$ Hz. The second and third harmonics of this fundamental will also give intensity maxima:

$$\begin{aligned} f_2 &= 2f_1 = 8,500\ \text{Hz} \\ f_3 &= 3f_1 = 12,750\ \text{Hz}\,. \end{aligned}$$

(The fourth and higher harmonics lie above the 15 kHz upper limit for the experiment.)

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8. **Photons; Decay; Estimation**

[1] (a) i. The work function is the energy required to remove a surface electron from a target metal.

[1] ii. The work function for copper is $\phi = (4.70)(1.6\times10^{-19}) = 7.52\times10^{-19}$ J.

2 iii. Photon energy is $E = hf = (6.64\times10^{-34})(3.0\times10^{15}) = 1.99\times10^{-18}$ J. From this, amount $\phi = 7.52\times10^{-19}$ J must be subtracted in order to overcome the lattice binding of the copper: $K_{\text{max}} = hf - \phi = (1.99 - 0.752)\times10^{-18} = 1.24\times10^{-18}$ J.

3 (b) The energy per photon is $hc/\lambda = (6.64\times10^{-34})(3\times10^8)/(550\times10^{-9}) = 3.62\times10^{-19}$ J. Therefore the number of photons required to register a flash is

$n = 1\times10^{-18}/(3.62\times10^{-19}) = 2.76 \approx 3.$

1 (c) i. A living 6-g sample should give (6)(16) = 96 decays/min.

2 ii. The expected count-rate after n half-lives is $N = N_0\left(\frac{1}{2}\right)^n$. Substituting $N = 10$ and $N_0 = 96$, then taking base-10 logs of both sides gives $\log_{10}(10/96) = n\log_{10}(\frac{1}{2})$. Solve for n to give $n = 3.26$ half-lives, corresponding to an elapsed time of $t = (3.26)(5700) = 18,600$ y.

1 (d) i. Escape speed for an Earth satellite is 11 km/s; this is also the final speed for a body, initially at rest, falling from infinity to Earth. (Meteor probably had a significant initial speed directed towards Earth.)

3 ii. Guess a spherical shape ($V = \frac{4}{3}\pi r^3$) with density ρ about, say, 4× that of water. Then its mass is m = density × volume $= \rho V = (4\times10^3)(\frac{4}{3}\pi 0.8^3) = 8579 \approx 1\times10^4$ kg (i.e., ~10 tonnes). Its kinetic energy would be $K = \frac{1}{2}mv^2 = \frac{1}{2}(1\times10^4)(16\times10^3)^2 = 1.38\times10^{12} \approx 1\times10^{12}$ J.

1 iii. For the train, $K_{\text{train}} = \frac{1}{2}mv^2 = \frac{1}{2}(10^6)(10^2)^2 = 5\times10^9$ J. That is, the meteor has about 200 times the kinetic energy of the train. Therefore one could anticipate very substantial damage had the meteor reached the ground!
