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# NZEST SCHOLARSHIP EXAMINATION 2001

# PHYSICS

## QUESTION AND ANSWER BOOKLET

Friday 9 November 2001, 2pm

Time allowed: Three hours

**Notes to Candidates**

1. The paper consists of:
   **10 short-answer questions** in Section A (worth 3 marks each)
   **8 long-answer questions** in Section B (worth 15 marks each).
   Full marks in Section B can be obtained by answering 7 questions, however all marks obtained from the 8 questions will count towards your final mark. The total mark is 135.

2. To get full marks, you will need to show your working and your reasoning. In addition, up to 8 **bonus marks** may be awarded in Section B (one mark per long question) for answers showing exceptional insight.

3. Work to 3 significant figures unless otherwise directed.

4. The standard list of Bursary Physics formulas is printed at the front of this book.

5. Answer all questions in this combined question/answer book.

## DATA WHICH MAY BE REQUIRED

| | | | |
|---|---|---|---|
| Acceleration of gravity | $g$ | = | $9.80\ ms^{-2}$ |
| Permittivity of free space | $\varepsilon_0$ | = | $8.85 \times 10^{-12}$ F $m^{-1}$ |
| Permeability of free space | $\mu_0$ | = | $4\pi \times 10^{-7}$ H $m^{-1}$ |
| Electronic charge | $e$ | = | $1.60 \times 10^{-19}$ C |
| Coulomb-law constant | $k$ | = | $8.99 \times 10^{9}$ N $m^2$ $C^{-2}$ |
| Planck's constant | $h$ | = | $6.63 \times 10^{-34}$ J.s = 4.14 |
| Speed of light | $c$ | = | $3.00 \times 10^{8}$ m $s^{-1}$ |
| Speed of sound in air | | = | 340 m $s^{-1}$ |
| Avagadro's number | $N_A$ | = | $6.02 \times 10^{23}$ $mol^{-1}$ |

**Physics Formulae reprinted courtesy of New Zealand Qualifications Authority**

$$F_g = \frac{GMm}{r^2}$$
$$F_c = \frac{mv^2}{r}$$
$$\Delta p = Ft$$
$$\omega = 2\pi f$$
$$d = r\theta$$
$$v = r\omega$$
$$a = r\alpha$$
$$F = ma$$
$$p = mv$$
$$v = v_i + at$$
$$v^2 = v_i^2 + 2ad$$
$$d = \frac{(v_i + v)t}{2}$$
$$d = v_i t + \tfrac{1}{2}at^2$$
$$\omega = \frac{\Delta\theta}{\Delta t}$$
$$\alpha = \frac{\Delta\omega}{\Delta t}$$
$$L = I\omega$$
$$L = mvr_\perp$$
$$\tau = I\alpha$$
$$\tau = Fr$$
$$E_{K(ROT)} = \tfrac{1}{2}I\omega^2$$
$$E_{K(LIN)} = \tfrac{1}{2}mv^2$$
$$\omega = \omega_i + \alpha t$$
$$\omega^2 = \omega_i^2 + 2\alpha\theta$$
$$\theta = \frac{(\omega_i + \omega)t}{2}$$
$$\theta = \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$
$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$E = \tfrac{1}{2}kA^2$$
$$a = -\omega^2 y$$

$$y = A\sin\omega t \qquad y = A\cos\omega t$$
$$v = A\omega\cos\omega t \qquad v = -A\omega\sin\omega t$$
$$a = -A\omega^2\sin\omega t \qquad a = -A\omega^2\cos\omega t$$

$$\Delta E = Vq$$
$$V = Ed$$
$$Q = CV$$
$$C_{TOT} = C_1 + C_2$$
$$\frac{1}{C_{TOT}} = \frac{1}{C_1} + \frac{1}{C_2}$$
$$E = \tfrac{1}{2}QV$$
$$C = \frac{\varepsilon_0\varepsilon_r A}{d}$$
$$\tau = RC$$
$$\frac{1}{R_{TOT}} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$R_{TOT} = R_1 + R_2$$

$$\phi = BA$$
$$\varepsilon = BAN\omega\sin\omega t$$
$$\varepsilon = -\frac{\Delta\phi}{\Delta t}$$
$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$
$$\varepsilon = -M\frac{\Delta I}{\Delta t}$$
$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$
$$E = \tfrac{1}{2}LI^2$$
$$\tau = \frac{L}{R}$$
$$I = I_{MAX}\sin\omega t$$
$$V = V_{MAX}\sin\omega t$$
$$I_{MAX} = \sqrt{2}I_{rms}$$
$$V_{MAX} = \sqrt{2}V_{rms}$$
$$X_C = \frac{1}{\omega C}$$
$$X_L = \omega L$$
$$V = IZ$$
$$n\lambda = \frac{dx}{L}$$
$$n\lambda = d\sin\theta$$
$$f = |f_1 - f_2|$$
$$f' = f\frac{V_W}{V_W \pm V_S}$$
$$E = hf$$
$$hf = \phi + E_K$$
$$E = mc^2$$
$$E_n = -\frac{hcR}{n^2}$$
$$\Delta E = |E_1 - E_2|$$
$$\upsilon = f\lambda$$
$$f = \frac{1}{T}$$

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## Section A: Guided Short-Answer [3 marks each]

*You must show your reasoning to gain the full three marks. Two marks are allocated for reasoning, one mark for making the correct choice (**T** or **F** for a True/False question; one of **a**, **b**, **c**, **d**, **e** for a multichoice question). A correctly chosen option without supporting working can earn no more than one mark.*

1. A rocket works by propelling some of its mass rearward. As the rocket's mass is always decreasing, Newton's second law of motion does not apply to the rocket. *True or false*?

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2. Two parallel wires each carry a current and therefore experience a force between them. When the direction of both currents is reversed, the direction of the force also reverses. *True or false*?

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3. When a child is standing on a swing the frequency of oscillation is $f_1$. When the child is sitting the frequency changes to $f_2$. Frequency $f_1$ is greater than $f_2$. *True or false*?

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4. A singly charged ion of helium ($He^+$) is smaller in size than a hydrogen atom. *True or false*?

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5. Reception of AM band (200 m < $\lambda$ < 600 m) radio waves is possible even if the receiver is not in direct line of sight of the transmitter. *True or false*?

6. A parallel plate capacitor has a potential difference $\Delta V$ between its plates. The plates are then pulled apart so that the separation between them is doubled while the charge on the plates is constant. The potential difference between the plates is now

(a) $\Delta V/2$ (b) $\Delta V$ (c) $2\Delta V$ (d) $4\Delta V$ (e) Some other value

7. A horizontal spirit level containing a bubble in a liquid is jerked forward. Relative to the level and liquid the bubble moves:

(a) backwards,
(b) forwards,
(c) not at all. The bubble and liquid move together.

8. For this question make the assumption that water is pushed from the base of a tree to its top by atmospheric pressure, about 100 kPa. If the water column in a tree was supported entirely by atmospheric pressure the maximum possible tree height would be about:

(a) 0.1 m; (b) 1 m; (c) 10 m; (d) 50 m; (e) 100 m.

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9. The power $P$ in Joule per second available from the wind for driving a windmill is given by an expression of the form:

$$P = \frac{1}{2}\rho A V^n$$

where $\rho$ is the density of the air, $A$ is the area swept out by the windmill's blades and $V$ is the speed of the wind. The correct value for the index $n$ is:

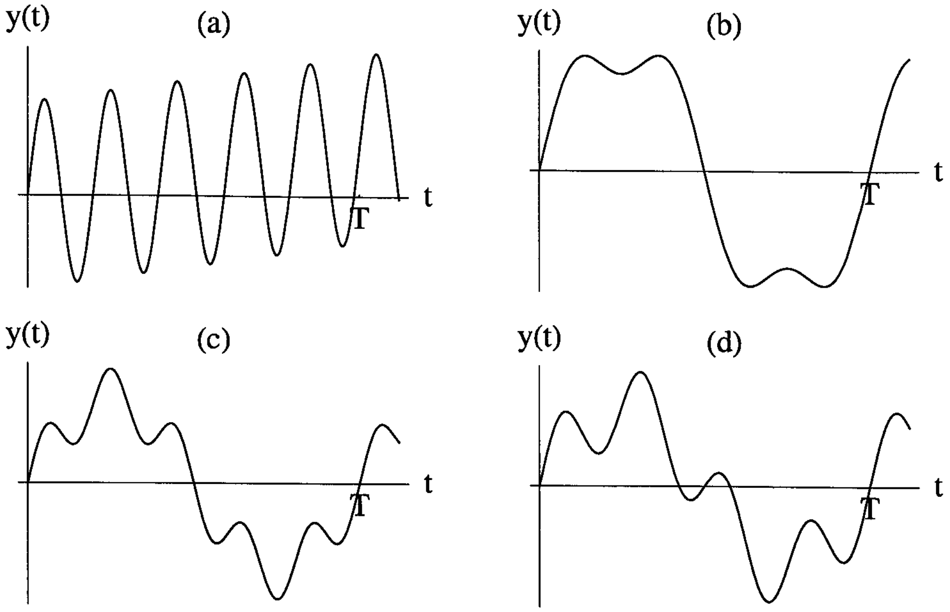
(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

10. A complex wave is represented by an expression of the form:

$$y(t) = \sin(\omega t) + \frac{1}{3}\sin(5\omega t)$$

where $\omega = \frac{2\pi}{T}$ is the angular frequency and $T$ is the period of the wave.

Which of the following sketches best represents the wave?



## SECTION B: LONG QUESTIONS [15 marks each]

## QUESTION B1:

Useful moments of inertia

| | |
|---|---|
| Solid cylinder | $I = \frac{1}{2}MR^2$ |
| Hollow cylinder | $I = MR^2$ |
| Solid sphere | $I = \frac{2}{5}MR^2$ |

Useful Data: Mean radius of Earth 6370 km
Mass of Earth $5.98 \times 10^{24}$ kg

(a) Precision measurements show that the length of Earth's day is increasing by $2.5 \times 10^{-9}$ seconds per day.

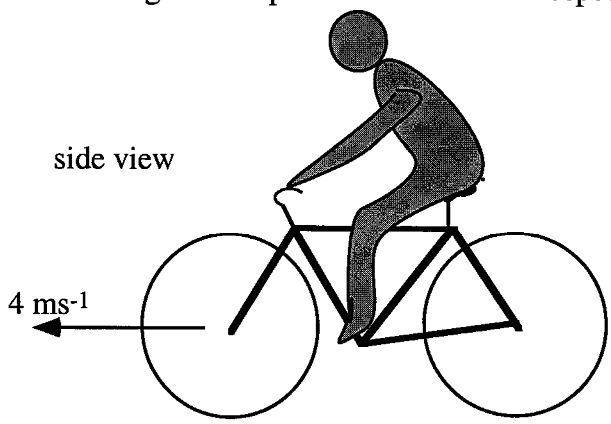
(i) Compute the length of a day one billion ($10^9$) years from now assuming the angular acceleration is constant. **[2 marks]**

(ii) Show that Earth's angular acceleration is about $-2.4 \times 10^{-23}$ rad s$^{-2}$. **[2 marks]**

(iii) Estimate the torque on Earth. **[2 marks]**

(iv) What is generating this torque? **[1 mark]**

(b) The diagram shows a woman on a bicycle. The total mass of the woman+bicycle is 75 kg. Each wheel has a mass of 1 kg, with most of that mass located on the rim of the wheel. The radius of each wheel is 0.4 m. She is travelling with a speed of 4 m $s^{-1}$ with respect to the road.

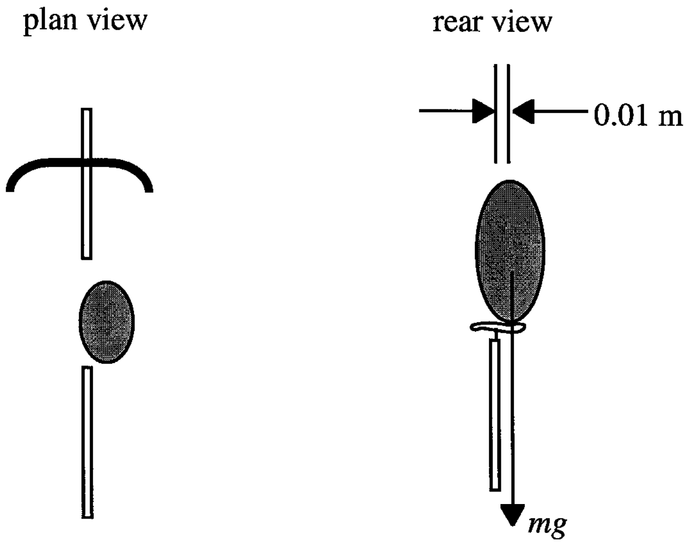



(i) Write down the angular speed of the wheels. **[1 mark]**

(ii) Calculate the angular momentum of each wheel. **[1 mark]**

(iii) Calculate the total kinetic energy of the woman+bicycle. **[2 marks]**

The woman is actually sitting on the seat a little too far (0.01 m) to one side (see diagram) and as a result she experiences a torque which tends to tip her over. The woman's mass is 70 kg.



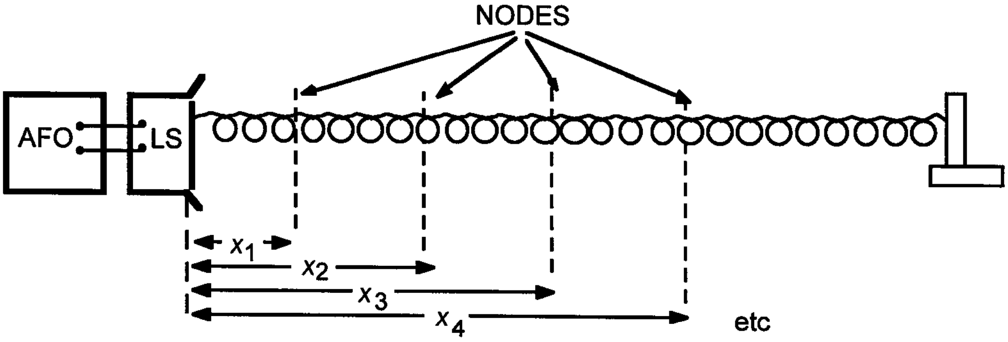

(iv) Calculate the torque which tends to tip the woman over. **[2 marks]**

(v) Calculate the work done by this torque if the woman tips by 3° ($\frac{\pi}{60}$ radian) **[1 mark]**

(vi) What can she do to compensate for this torque? **[1 mark]**

## QUESTION B2:

In an experiment to study longitudinal standing waves on a spring, a small light spring was stretched horizontally between a rigid clamp and the diaphragm of a loudspeaker. The loudspeaker was connected to an audio frequency oscillator and as the frequency was varied a series of standing waves was produced. In each case where a clearly defined standing wave was produced, the positions of the displacement nodes were measured from the point of attachment of the spring to the diaphragm as shown.




These distances and the corresponding frequencies were tabulated as below.

Stretched length of spring = 21.0 ± 0.1 cm.

| Frequency (Hz) ±5 Hz | Positions of nodal points (cm) ± 0.1 cm |
|---|---|
| 290 | 6.8  13.8 |
| 390 | 5.3  10.5  15.6 |
| 490 | 4.3  8.4  12.6  16.7 |
| 580 | 3.4  7.0  10.5  13.8  17.3 |

(a) On theoretical grounds, what would be expected at the rigid clamp at 21.0 cm? **[1 mark]**

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(b) What can be inferred from the tabulated nodal points about the diaphragm at 0.0 cm? Does it behave very nearly as a displacement node, as an antinode, or somewhere in between? **[1 mark]**

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(c) Using the answers to (a) and (b) as well as the tabulated values, complete the following table of frequency and corresponding average internodal distance. **[4 marks]**

| Frequency | Average internodal distance |
|---|---|
| | |
| | |
| | |
| | |

(d) State the relationship between the internodal distance and the corresponding wavelength. **[1 mark]**

(e) Using this relationship, obtain values for the speed of the longitudinal wave in the spring for each frequency. Calculate the average speed over this range of frequencies. **[3 marks]**

(f) Using the values of the wave speed calculated in (e), suggest an uncertainty in the average speed. **[1 mark]**

(g) The behaviour of the spring was next examined using a stroboscope. When the stroboscope was run at very nearly the same frequency as the waves, the movement of the spring was seen in slow motion.

Describe the variation in motion of the portion of the spring between any two nodes as seen using the stroboscope light. **[2 marks]**

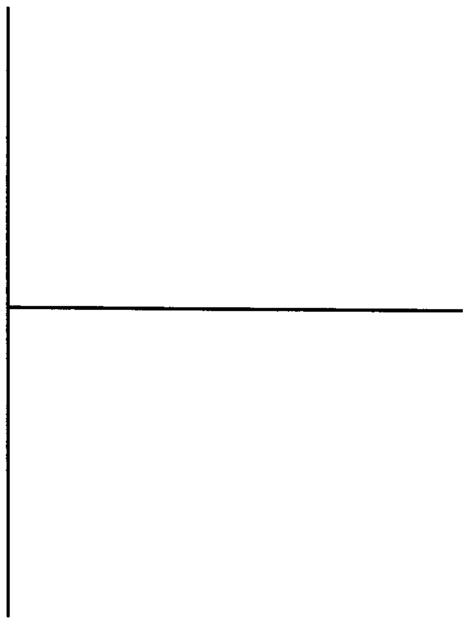
(h) Longitudinal waves in a spring are very similar to longitudinal sound waves in a column of air.

Draw a labelled diagram of an air column arrangement closely analogous to the spring arrangement, showing the possible positions of the displacement nodes. **[2 marks]**

## QUESTION B3:

A light industrial factory is connected to the NZ mains supply of 230 V (rms) at 50 Hz. The factory draws 120 A (rms) from the supply. However the load is very inductive and the supply voltage and the current are out of phase by 40°.

(a) Using the skeleton below, draw a phasor diagram for this situation, including the magnitudes and components of all phasors. **[5 marks]**



(b) Explain why the power drawn by the factory is less than 27.6 kW (230 V × 120 A). **[2 marks]**

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To bring the supply voltage into phase with the current, a consultant proposes to connect a large capacitor in series with the factory's load.

(c) Determine the capacitance of the capacitor required to make the phase difference 0°. **[2 marks]**

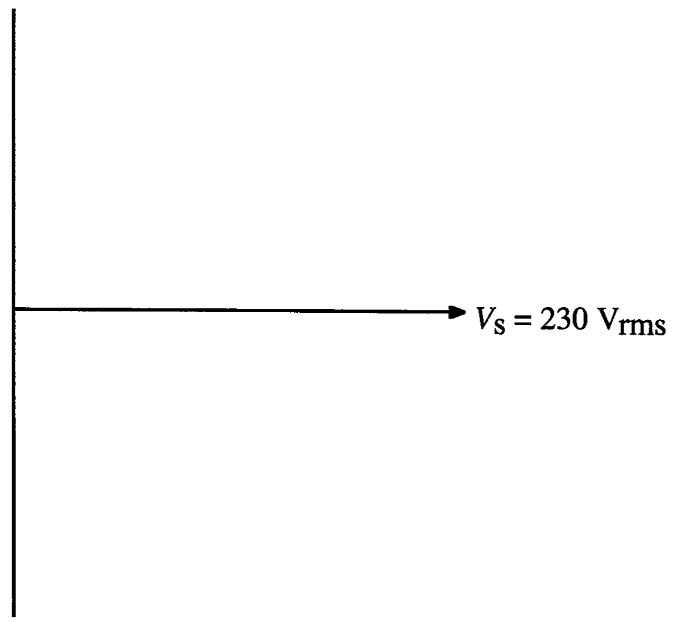
(d) With the capacitor in series with the load, find the rms potential difference across the original load. **[1 mark]**

A second consultant suggests that a better solution would be to connect a capacitor in parallel with the load.

(e) Apart from being able to be installed without breaking the circuit, suggest one advantage of connecting a capacitor in parallel. **[1 mark]**

(f) Sketch a new phasor diagram for the circuit with the capacitor connected in parallel with the load and with the phase difference 0°.

(Hint: Each branch of the circuit has voltage and current phasors. Which of these is the same for both branches?) **[4 marks]**



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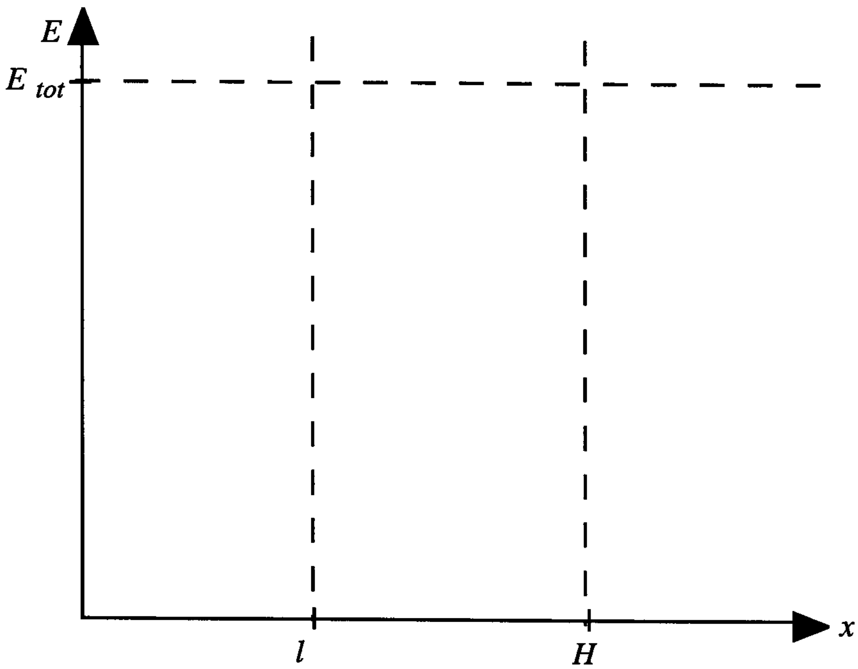
## QUESTION B4:

Bungy jumping is a sport in which a person jumps from a high platform using strong elastic bands which have one end tied to the jumper's ankles and the other end (firmly!) attached to the jumping platform. The length of the bands is adjusted so that the jumper just touches the surface of the water below the platform before the bands pull them upwards.

Let the mass of the jumper be $m$, the height of the jumping platform above the water be $H$, and the unextended length of the elastic bands be $l$ ($l < H/2$).

(a) (i) Using the plot below, sketch and label the gravitational potential energy (GPE), the elastic potential energy (EPE), and hence the kinetic energy (KE) of the jumper as a function of $x$, the distance fallen by the jumper. **[3 marks]**




(ii) Mark on your graph the position of maximum kinetic energy. **[1 mark]**

(b) Using conservation of energy at the position of maximum stretch, show that the spring constant of the elastic bands is given by **[3 marks]**

$$k = \frac{2mgH}{(H - l)^2}$$

(c) Explain why the jumper has maximum downward speed when the acceleration is zero. **[1 mark]**

(d) Show that the jumper's maximum speed is given by **[3 marks]**

$$v_{max}^2 = 2gl + \frac{mg^2}{k}$$

(e) Find the maximum upward acceleration of the jumper. **[2 marks]**

(f) Give an argument that the spring constant of the elastic bands is inversely proportional to the length of the bands. **[2 marks]**

## QUESTION B5:

Radiocarbon dating is a useful technique for finding the age of materials, particularly those of organic origin. The key to radiocarbon dating is the radioactive properties of the carbon isotope $^{14}C$.

**Note:** In this question you should work to as many significant figures as necessary but your answers should be expressed with only 3 significant figures.

**Useful Data:**

| | |
|---|---|
| Half-Life of $^{14}C$: | 5730 year |
| Mass of electron: | $9.109 \times 10^{-31}$ kg |
| Atomic weight of carbon: | 12.011 gram mol$^{-1}$ |

Atomic masses of some isotopes (all in units of $10^{-27}$ kg):

| | | | | | |
|---|---|---|---|---|---|
| $^{12}C$ | 19.926 480 | $^{13}C$ | 21.592 583 | $^{14}C$ | 23.252 943 |
| $^{13}B$ | 21.616 544 | $^{13}N$ | 21.596 535 | $^{14}N$ | 23.252 665 |
| | | | | $^{14}O$ | 23.261 836 |

$^{14}C$ decays via beta decay where a neutron in the nucleus decays into a proton and ejects an electron.

(a) For the decay reaction below, fill in the gaps to describe the decay product. **[4 marks]**

$$^{14}_{6}C \rightarrow {}^{0}_{-1}e^{-} + {}^{A}_{Z}X^{C}$$

A(mass number): ____________________

Z (atomic number): ____________________

C (decay product charge): ____________________

X (decay product symbol): ____________________

(b) Given that the initial $^{14}C$ atom is neutral, determine the total mass of the decay products (you should ignore electron binding effects). **[1 mark]**

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(c) Show that the energy released per $^{14}C$ decay is $2.50 \times 10^{-14}$ J. **[2 marks]**

(d) The law of radioactive decay is

$$\frac{dN}{dt} = -\lambda N$$

where $N$ is the number of radioactive atoms present at a given time and $\lambda$ is a constant related to the half-life.

(i) Write down the units of the left-hand side. **[1 mark]**

(ii) What does the negative sign indicate? **[1 mark]**

(iii) Explain what this equation means. **[1 mark]**

(e) The relation between the constant $\lambda$ and the half-life is $\lambda = \frac{0.693}{T_{1/2}}$

Calculate $\lambda$ for $^{14}C$ in $s^{-1}$ **[1 mark]**

Before 1900 the activity of atmospheric carbon per gram of carbon due to $^{14}C$ was (on average) 0.255 decays per second. The ratio of $^{14}C$ to total carbon in the atmosphere is (to a good approximation) constant.

(f) Using the half-life of $^{14}C$, show that this activity corresponds to about $6.65 \times 10^{10}$ atoms of $^{14}C$ per gram of carbon. **[2 marks]**

Living organisms have the same proportion of $^{14}C$ as the atmosphere. However when an organism dies it stops taking in carbon from the atmosphere, and the $^{14}C$ in the organism decays. We can use this to determine the approximate date of death of an organic archeological specimen.

(g) A specimen of mammoth bone is found to have a $^{14}C$ activity of 0.043 decays per second per gram of carbon. Assuming that when the mammoth lived the activity of $^{14}C$ was the atmospheric average value of 0.255 decays per second per gram of carbon, determine the age of the specimen.

**[2 marks]**

## QUESTION B6:

One of the fundamental features of quantum mechanics is wave-particle duality. Both light and matter can show either wave or particle aspects in appropriate experiments. In particular, diffraction of atoms has been experimentally observed. In these experiments Louis de Broglie's hypothesis that the wavelength $\lambda$ of a particle is given in terms of its momentum $p$ and Planck's constant $h$ by,

$$\lambda = h/p$$

was verified.

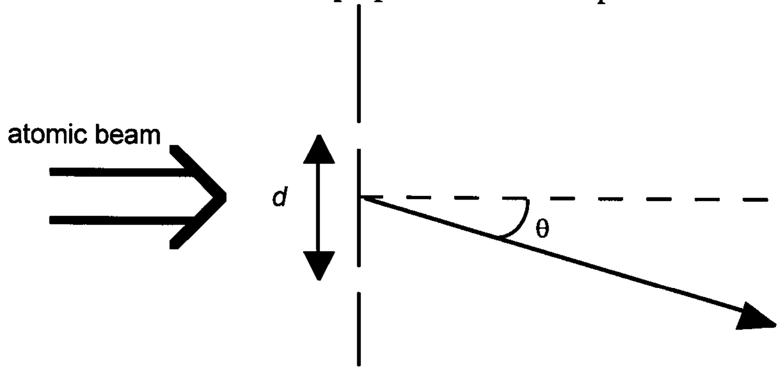
(a) Show that the de Broglie wavelength $\lambda$ for an atom with kinetic energy $E$ and mass $m$ is

$$\lambda = \frac{h}{\sqrt{2mE}}$$ **[2 marks]**

(b) Very cold ($T \sim 1\mu$K) sodium atoms have a kinetic energy of about $1.2 \times 10^{-10}$ eV. Estimate the de Broglie wavelength of these atoms **[2 marks]**

**Data:** Atomic mass of sodium $38.2 \times 10^{-27}$ kg

Consider a Young's type experiment for atoms. A beam of atoms is incident on a double slit with slit separation $d$. The beam is well collimated and perpendicular to the plane of the slits (see figure below).




The interference pattern is observed at a large distance from the slits.

(c) Derive expressions in terms of $E$ for the angles of the intensity maxima and minima of the atomic interference pattern **[4 marks]**

One experimental difficulty is that the incident atoms have a range of energies. Assume that they have a range of energies uniformly distributed between $E + \Delta E$ and $E - \Delta E$, so that the mean energy is $E$. Because of the range of energies the experimental diffraction fringes are poorly defined compared to the ideal single energy case.

(d) Explain why having a range of energies in the atomic beam leads to poor definition of the experimental interference fringes. **[2 marks]**

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A condition for observing the interference fringes is that all first order minima must occur at smaller angles than any first order maxima.

(e) Give an expression in terms of $E$ and $\Delta E$ for $\theta_a$ the largest angle at which a first order minimum can occur. **[1 mark]**

(f) Give an expression in terms of $E$ and $\Delta E$ for $\theta_b$, the smallest angle at which a first order maximum can occur. **[1 mark]**

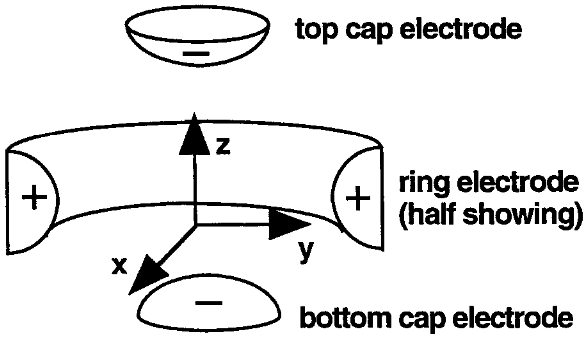
(g) Show that to satisfy the condition for observing the interference fringes we require

$$\Delta E < \frac{3}{5} E$$

**[3 marks]**

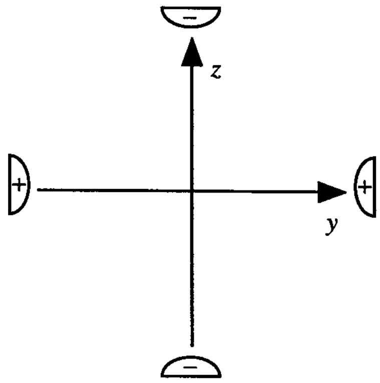
## QUESTION B7:

The Penning trap is a device for trapping individual electrons for long periods of time. The trapping is provided by the combined effect of electric and magnetic fields. A schematic diagram of a Penning trap is shown in the figure. The coordinate origin is in the geometrical centre of the trap.




(a) Sketch below the electric field lines in the vertical ($y$, $z$) plane.

**[2 marks]**

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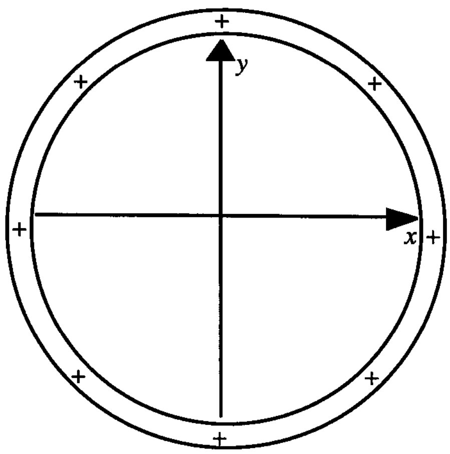
(b) Sketch below the electric field lines as viewed from above. **[2 marks]**

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(c) Along the $z$ axis the electric field due to the electrodes is $E = E_o z$ and is parallel to the $z$ axis.

(i) Describe (in words) the motion of an electron along the $z$ axis, giving reasons for your description. **[3 marks]**

(ii) Give an expression for the frequency of the electron's motion and the numerical value of this frequency for $E_o = 2 \times 10^3$ V m$^{-1}$. **[3 marks]**

(iii) Describe (in words) the motion of an electron that is not on the $z$ axis (assume only an electric field is present). **[1 mark]**

(d) Explain why a strong magnetic field parallel to the $z$ direction is useful for confining electrons in the radial direction (perpendicular to the $z$ direction). **[2 marks]**

(e) When the magnetic field is strong enough the radial electric field can be ignored and the electrons undergo circular 'cyclotron' motion.

The magnitude of the magnetic force on an electron is $F = Bev$. Show that the cyclotron frequency is

$$f = \frac{Be}{2\pi m}.$$

**[2 marks]**

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## QUESTION B8:

(a) Consider a body of mass $M$ and initial velocity $V$. In order to lose energy the body emits a particle with energy $E$ and momentum $p$ parallel to the initial velocity $V$. The mass of the particle is much less than $M$ and can be ignored in this problem.

(i) If the final velocity of the body is $V'$, write down equations expressing the conservation of energy and momentum in the process.

**[2 marks]**

(ii) By eliminating $V'$, show that the energy and momentum of the emitted particle must be related by

$$E = Vp - \frac{1}{2M} p^2$$

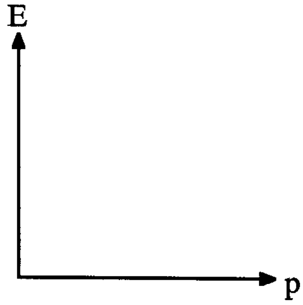
**[2 marks]**

(iii) Using (ii), show that for $M$ very large there is a lower limit on the velocity $V$ of the body (i.e. if the velocity of the body is smaller than this limit, it cannot emit a particle with energy $E$ and momentum $p$).

$$V \geq \frac{E}{p}$$

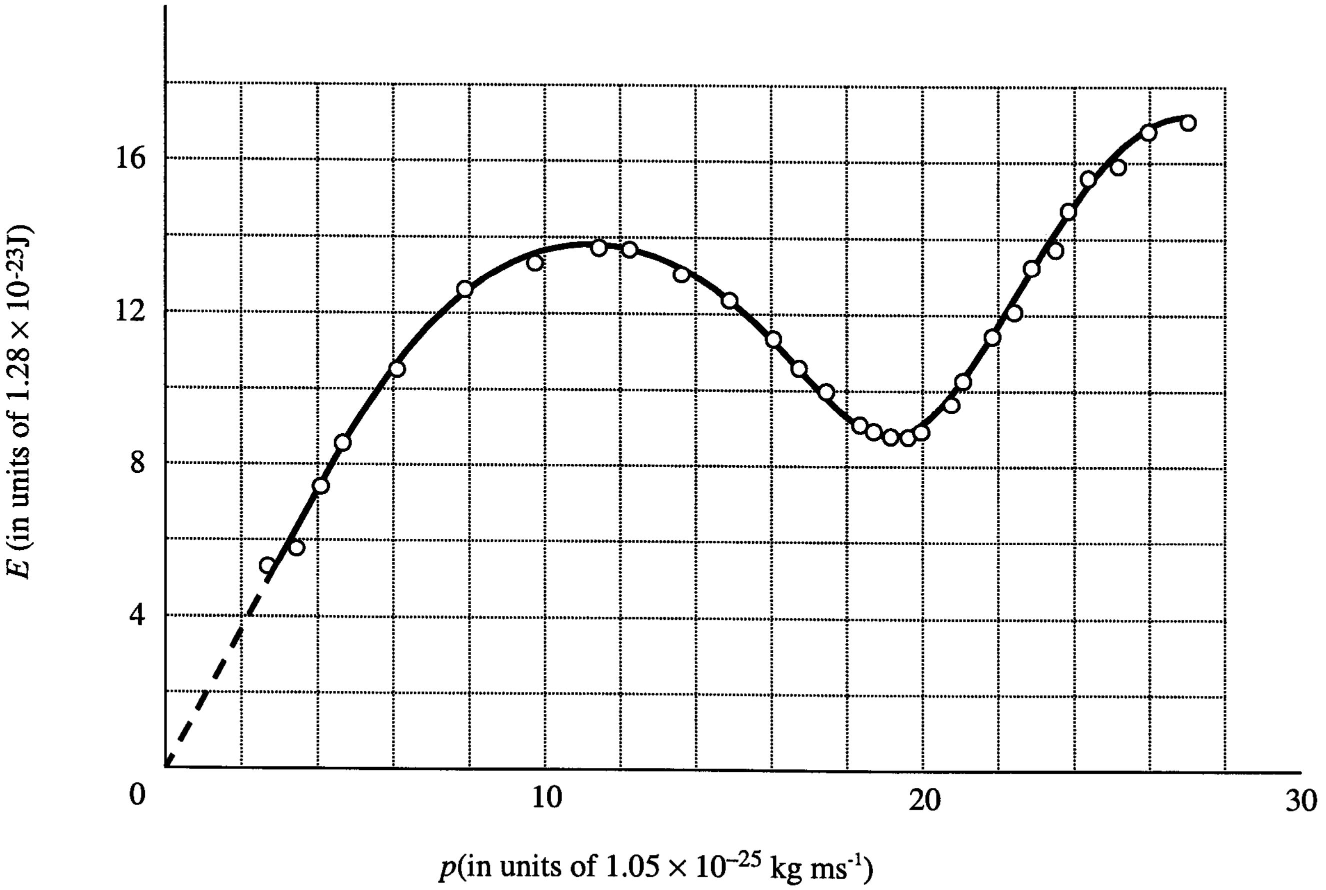
**[1 mark]**

(iv) If the emitted particle is an atom of mass $m$, sketch a plot of its kinetic energy against its momentum. **[2 marks]**




(b) It is often impossible for a body moving in a fluid to emit atoms (or other massive particles). In some cases the only way for the body to lose energy is to emit particles which are massless and hence quantum mechanical in nature. Such particles are called (collectively) excitations.

One such case is a body moving in liquid $^4He$ at temperatures below 2.17 K. Shown below is the experimentally determined 'dispersion curve' for liquid $^4He$, giving the relation between $E$ and $p$ for any excitation emitted by a body moving in the liquid.



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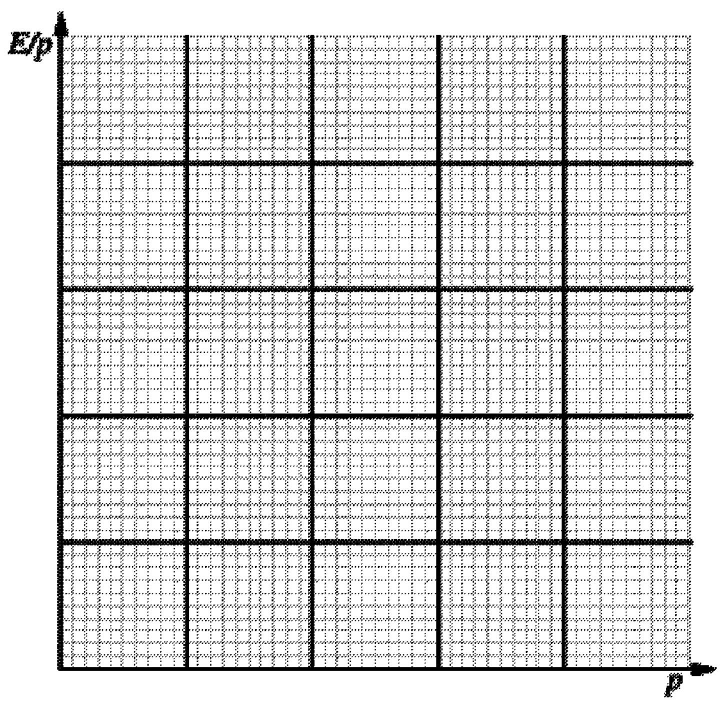
(i) Write down two features of the dispersion curve for liquid $^4He$ that are different from the plot of $E$ vs $p$ you sketched in part (a)(iv) for a classical particle. **[2 marks]**

(ii) From the dispersion curve for liquid $^4He$, evaluate the ratio $(E/p)$ (in m $s^{-1}$) at $p$ = 5, 10, 15, 20, 25 (in the units plotted on the dispersion curve), and sketch a plot of $(E/p)$ vs $p$. **[4 marks]**



(iii) Estimate the minimum value of $(E/p)$ for excitations in liquid $^4$He. **[1 mark]**

(iv) In part (a)(iii) you showed that the velocity of a body generating the excitations satisfies $V \geq E/p$. Comment on what happens to the body if its velocity is smaller than the minimum value of $(E/p)$ you found in part (b)(iii). **[1 marks]**

-End of Examination-