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NZEST SCHOLARSHIP EXAMINATION

1999 EXAMINER'S REPORT AND SOLUTIONS

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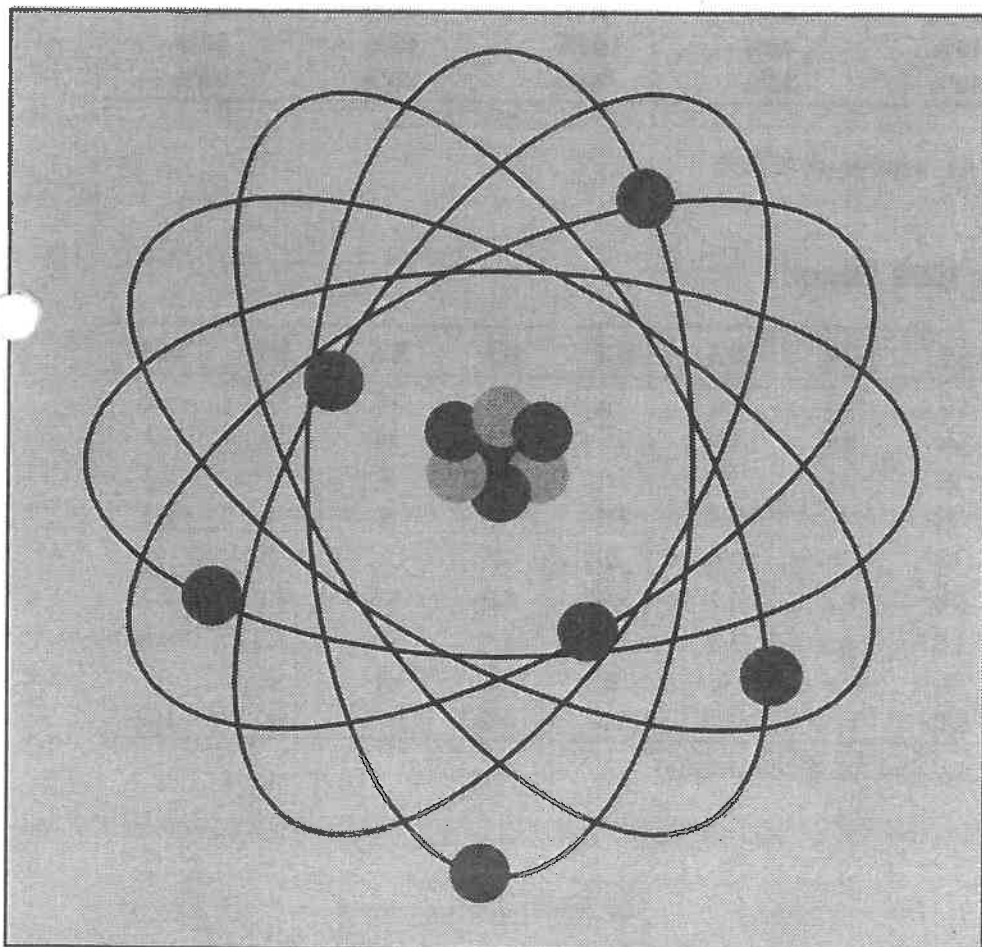
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1999 PHYSICS EXAMINER'S REPORT**Contents:**

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I. Exam Statistics & Comparison with previous years

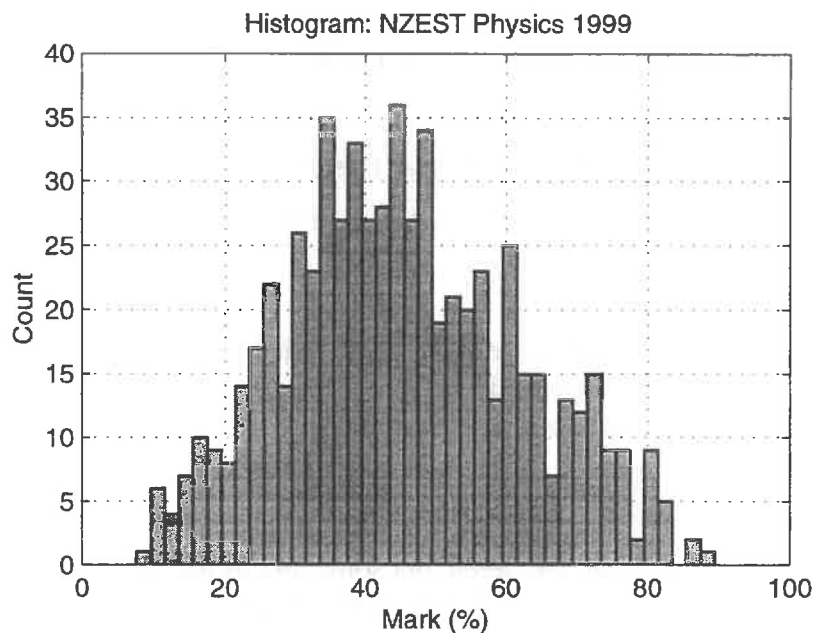
	1999	1998	1997	1996	1995
No. of students	642	659	596	595	481
Top score	89%	91%	89%	86%	99%
Upper quartile	57%	61%	54%	53%	80%
Median	45%	49%	44%	42%	53%
Lower quartile	33%	38%	34%	30%	38%

19 students were graded A+ with mark $\geq 79\%$

II. Question Statistics for 1999 Paper

	A	B1	B2	B3	B4	B5	B6	B7	B8
Max possible	30	15	15	15	15	15	15	15	15
Top*	31	15	16	14	15	15	14	16	15
Upper qu.	21.5	9	11	8	10	9.5	8	7	8.5
Median	17	7	9	5.5	7.5	6	5	4	5.5
Lower qu.	13	4	7	3.5	5	2	2	1	2
Mean	17.4	6.8	8.8	5.7	7.4	6.3	5.0	4.6	5.7
Std devn	6.1	3.5	3.0	3.1	3.6	4.5	3.8	4.3	4.0
Bottom	1	0	0	0	0	0	0	0	0
N	642	642	642	642	642	642	642	642	642

* One bonus mark was available for each question



III. General Comments from the Examiner

This is my second year as Physics Examiner. I have maintained the same exam format as for 1998, but reduced the number of long questions from nine to eight, since last year's paper seemed a little long. I had hoped this shortening would serve to raise the median slightly from last year's 49%, but this didn't happen: this year the median came in at 45%. I've received numerous strong statements from physics teachers to the effect that the **standard** of the scholarship paper should *not* be lowered; instead candidates should rise to the challenge which this exam presents. (See section VI.5 towards the end of this report.)

Last year I introduced the notion of a **bonus mark** in Section B for answers showing real insight and talent. This year the marking panel decided to extend the eligibility for bonus marks to Section A as well. Thus, in principle, a student could gain a total of nine bonus marks (1 bonus in Section A plus 1 bonus for each question in Section B). The intent is to encourage students to put in the extra effort to craft truly excellent responses. This incentive will be retained for the 2000 exam.

Some people have suggested that the scholarship paper should include a list of **reference formulas**, much as is done for bursary. My personal preference is to maintain the status quo of not having a formula sheet, but am happy to be guided in this. It is my intention to survey the opinions of schools on this matter early in the year — *if* there is a strong feeling that scholarship physics students should be issued with a formula sheet, then this can happen for the 2000 exam. Please ensure your school responds when the questionnaire from NZEST arrives.

I would like to remind candidates that the scholarship physics exam is designed to *test general thinking in physics, probing student understanding of basic physics concepts and skills just a little beyond the strict confines of the bursary prescription* (in a horizontal rather than a vertical sense). These "**basic physics concepts**" certainly include material which is part of the form-6 physics prescription. My advice to candidates is that as part of your preparation for the scholarship physics exam, you should revise the form-6 (level-7) material you were taught in your form-6 year. You should also be familiar with log/log and semilog graphs to interpret data, know how to do order-of-magnitude estimation, and understand the notion of dimensional

consistency of physical equations. (For an example of an equation which is not dimensionally consistent, see the Aldiss version of $E = mc^2$ in question B8 of the 1999 paper.)

IV. Question-Specific Comments by Marking Panel

This section is a summary of comments made by members of the 19-person marking panel.

A. Guided Short Answer

- A1. Weightlessness: Generally answered well, but some failed to mention the absence of reaction force from the 'floor'. Some entertainers: "The force of gravity does not exist outside the atmosphere," and, "The astronauts are weightless because of the vacuum of space"(!)
- A2. Work by spring: Most made correct multichoice selection, but few came up with correct justification based on $W \propto x^2$.
- A3. RC time constant: Generally well answered, but some told us that the time-constant is the time required for the voltage to drop *by* 37% (RC is the time to drop *to* 37% of original, so the voltage has dropped by 63% in this time).
- A4. Bar magnet: Done well. Some sophisticated answers mentioned that the discovery of magnetic monopoles would mean a change to one of Maxwell's laws ($\oint \mathbf{B} \cdot d\mathbf{A} = 0$), since the magnetic flux through a closed surface surrounding a monopole would be non-zero!
- A5. Induced emf: Although many answered True, they often failed to state that the induced emf depends on the *rate* of change of magnetic flux.
- A6. Pendulum bob: Generally well done, though some students obtained an expression for F rather than the tension.
- A7. Rotational inertia: Straightforward question, done well by those who understood how to sum up the mr^2 contributions.
- A8. Pulse reflection: Some candidates failed to recognize that the leading edge would be reflected first.
- A9. LC circuit: Many failed to state that the signal generator *drives* the circuit, hence determines the circuit frequency. The fact that the current and voltage will be out of phase has no bearing on circuit frequency. One student told us: "The 50 Hz and 16 Hz sources will create a beat frequency causing constructive and destructive interference"(!)
- A10. Radioactive decay: Most were able to establish that 1.5 half-lives had elapsed, but fewer could give a mathematical justification for their final answer. Many students could not manage the $480 \times (1/2)^{1.5}$ calculation.
- *General comment:* This Section A style of justified short-answer sorts the physics students out precisely. A bonus mark was awarded to the best scripts.

B1: Floating, Falling, and Log-log Skills

- (a) A common error was to claim that when the bowl moved beyond the edge, water was displaced to the left, thus moving the centre of mass to the left, making the system even more stable! (If this were the case, then the system would be unstable in its initial configuration...)

- (b) Very few could give a complete description equating gravitational force to ma to earn full marks; most who were on track got 2 out of 3.
- (c) Generally well done, despite the fact that about half the students did not attempt the log-log graph. Of those who did, many could not cope with the x -values spanning more than a single cycle of the graph.

B2: Plants, Plumb-lines, Candles & Planes

- (a) Some confusion about accelerated frames of reference. Little understanding that circular motion is an accelerated motion (because the velocity vector is continuously changing direction) requiring an unbalanced inwards force. For the candle flame, it may be helpful to think of this as similar to a buoyant plumb-line (e.g., a tethered helium balloon) which will incline inwards.
- (b) Some discerning students pointed out that at the moment before lift-off, the aircraft nose rotates upwards, so that the $a = g \tan \theta$ relationship which applied during the acceleration phase will no longer be true.

• *Marker comment:* A good discriminating question, but with a pleasingly high median of 9.

B3: Statics; SHM (tides); Satellites

- (a) (i) Very few students included the reaction force of the hinge. Some showed T as a vertical force or located the other forces incorrectly, though this did not always affect their answers to part iii. (ii) Few realized that with this choice of origin, R would be eliminated from the moments equation. (iii) Often students wrote down the force-balance principle ($\Sigma F_{\text{up}} = \Sigma F_{\text{down}}$) as a justification of applying moments balance. Some forgot to include g . Others had problems with vector resolution.
- (b) Some students made work for themselves by attempting to extract values from the graph rather than using the values listed in the table. (i) About a third of the students doubled the time between high and low tides; the better students took an average of values over two cycles. (ii) Determining the amplitude of an offset sinewave presented difficulties for about a quarter of the candidates, as did calculating the time difference required in part (iii). In (iv), many students did not show their full working.
- (c) Few gained the full 2 marks here. While about a third recognized the need for the satellite to be above the Equator, many got distracted by the notion of ‘parking’ the satellite and were not able to extract the essential physics.

B4: Gravity and Electricity

- (a) The value of G was omitted from the paper—this was an oversight for which the Examiner apologises. We made allowance for this omission in the marking.
- (b) Too many candidates think “calculate the ratio” means “replace the symbols with numbers but don’t bother doing any calculation”! In (iii), many answers showed a lack of understanding about the properties of electrostatic charge; a common suggestion was that planets have no charge. One student told us: “The Earth is, well, Earth, so by definition is uncharged!”
- (c) Candidates lost a mark if they failed to label their sketch graph for the charging curve. Many students relied on their knowledge of RC circuits to justify their choice of curve. Few showed

insight into the linear delivery of charge via the belt, or how this might change as the repulsive force on the belt increases. [Teachers might like to discuss with their students the distinction between charging a capacitor from a constant-voltage source versus charging from a constant-current source.]

B5: Resistors, Capacitors, and AC Circuits

- (a) Many students had difficulty converting the given information into a pair of simple circuits involving series resistors, probably because they failed to realize that the end that was *not* being measured was open circuit, so any resistances in those branches could be ignored. Those students who did produce correct diagrams usually gave correct equations which they were able to solve for R and x .
- (b, c) Mostly done well. The most common errors arose from not being able to add fractions properly! These people would then often end up with an answer which was dimensionally inconsistent (e.g., $R_{\text{tot}} = 2/R_0$), and would fail to recognize that their answer made no sense.
- (d) (i) Some calculated f_0 rather than the angular frequency ω_0 asked for. Both answers were accepted as correct. (ii) Most realized that at resonance $V_R = V_{\text{supply}}$, but then incorrectly determined the impedance and hence current in the circuit. A very common misconception is that at resonance, both V_C and V_L are individually zero. (iii) Most drew correct phasor diagrams, but about a quarter gave incorrect explanations.

B6: Magnetic Force, Magnetic Flux, and Induced emf

- (a) Very few candidates scored 2 marks. While a good number provided $\phi = BA$, few realized the need for the area normal to be parallel to the magnetic field. One good student showed an appreciation of the vector nature with a flux definition of $\phi = \int \mathbf{B} \cdot d\mathbf{A}$.
- (b) Most scored only 1 of the 3 marks available with variations on the theme of relative translation between magnet and wire. Rotation was sometimes given, and area change very seldom. Perhaps the question should have stated: "Give three *distinct* methods of emf induction."
- (c) (i) and (ii) Well done by most candidates. The most common error was failure to convert from cm to metres! (iii) Many correct responses for induced voltage and current, but current direction was frequently an imprecise "up" instead of the required anticlockwise. (iv) Surprisingly, most were able to correctly identify that $\mathcal{E} = 0$ with correct justification.
- (d) (i) and (ii) are easy Form-6 work. Done reasonably well although a number of students tried to use $F = BI\ell$ instead of $F = Bev$. (iii) Describing the path as "curved" was not adequate—we were looking for "circular".
- *Marker comment:* This electricity question appeared to be very straightforward, but was very disappointingly done giving a low median of 5.

B7: Waves and Interference

- (a) (i) Too few realized that they needed to describe the path difference as being an integral number of wavelengths to get constructive interference; most simply gave the meaning of each symbol. (ii) & (iii) Generally well done.
- (b) Well done.

- (c) (i) Most people failed to note that $d \neq 0$, but generally well done. (ii) Most failed to complete this part of the question. Those who got this far had little trouble manipulating the formula into a form suitable for graphing. The usual choice was to plot $(\cos\theta_{\text{in}} - \cos\theta_{\text{out}})$ vs m (or vice versa) rather than the form given in the model answers. The graphs were generally well done. A few candidates tried plotting θ_{out} vs m but earned no marks. Quite a few calculated d from one set of values (earning 1 mark); others calculated for a range of values and averaged them (earning 2 marks).
- *Marker 1 comment:* Personally, I felt that this question is primarily of Form-6 level and should have been done better. Only a small proportion of candidates scored close to full marks.
 - *Marker 2 comment:* A nice question which scholarship students should have been able to handle easily, though it was in fact poorly answered on the whole. Perhaps some candidates were short of time by the stage they reached this question.

B8: Nuclear & Atomic Physics; Estimation

- (a) About 50% of candidates understood this question and made the correct deduction. Very few used dimensional analysis—this concept is not well appreciated. One memorable response: “In nuclear reactions we are only concerned with objects moving at speeds close the speed of light. Since nuclear power stations do not move that fast, there will be little or no effect on them.”(!)
- (b) About 10% continued to use the fictitious exponent (2.7713) in their calculations; we deducted a mark. About 20% failed to use the accurate constants given (they used 3 and 1.6), so lost a half-mark for each. Many students handled “show that” poorly, often lacking a clear logical process in their arguments. Nevertheless, over half of the students scored 2 marks.
- (c) (i) A surprisingly high number knew about positrons! We awarded 1/2-mark for ${}^0_1\text{p}$ or ${}^0_1\text{e}$, full mark for positron or β^+ . (ii) This part was not so well done. Much confusion about atomic mass units, mass, and energy. Maths involving powers of 10 causes trouble for many. About a third forgot to convert mass from u to kg.
- (d) Lasers are not well understood, nor are the various properties of light. For the four qualities listed in the model answers, we awarded $\frac{1}{1/2} \frac{2}{\checkmark} \frac{3}{\checkmark\checkmark} \frac{4}{\checkmark\checkmark}$. Only about 20% of students gave three or more of the expected properties.
- (e) In contrast, many candidates coped well here, with a pleasing number scoring the full 5 marks.

V. Action Item Requests from the Marking Panel

- In the back flap of the answer book, pre-print the question numbers to avoid confusion when recording marks.
- We appreciated the comfortable chairs — much better than the wooden lab stools we sat on for three days in 1998! [**Examiner:** Thank you to Alex Binnie for organizing this]

VI. Survey Responses

Several senior physics teachers were surveyed for their appraisal of the exam. Here is a summary of their responses.

1. Was the Scholarship examination at the right level, i.e., sufficiently challenging, to extend the able student?

Five respondents said “yes” or “about right”. Other responses:

- perhaps too challenging given current population doing scholarship
- excellent paper with imaginative challenging questions which tested student’s conceptual understanding—possibly a bit hard?
- some good questions to challenge the more able
- yes — spot on!
- appeared to be straightforward but students did not score highly
- adequate
- only just [sufficiently challenging]!
- seemed a bit on the easy side

2. Were there any questions considered to be inappropriate?

Two respondents gave an unqualified “no”. Other responses—

- A7: “inertia” should have read “rotational inertia” [Examiner: Agree]
- A8 (reflection of a pulse on a string) [Examiner: This is form-6 concept, therefore a fair target for schol questioning]
- B1(a) (floating) is not in the prescription [Examiner: Flotation is an early junior science concept, therefore a legitimate target for schol questioning]
- No, apart from mistake of not providing the value for G in B4(a)(i) [Examiner: My apologies for this omission]
- B4(c)(i) inappropriate (charging of van de Graaff generator)
- B4 was marginal in parts
- B4: Examiner tried too hard to give a practical context—discussion of point masses/charges would have been sufficient
- No, but B5(a) was ambiguous as to whether total length of cable was 10 km or 20 km
- B6(d) (direction of magnetic force on electron) is not in the prescription [Examiner: Topic does appear in Form-6 prescription]
- B7: Bursary uses the formula $d \sin\theta = n\lambda$ (schol paper quoted $d \sin\theta = m\lambda$)
- B8: The 5 marks for life and tread-thickness of a tyre could have been better used.
- B8: Similar to last year’s last question, but further removed from accessible empirical knowledge of ink-in-pen. But a good question for testing those with real talent.

3. Was the coverage of the syllabus satisfactory?

Six respondents gave an unqualified “yes”. Other responses—

- Yes — even coverage
- Yes — dipped into Form-6 work as well which was a good test of whole-subject learning

VII. Acknowledgements

My sincere thanks to Ian Walker and Alex Binnie who moderated the paper; to the marking panel of senior physics teachers; to university colleagues Lara Wilcocks and Michael Cree who agreed to join me on the marking panel for a second year; and to the computing staff at St Kentigern's College who ensured that the temporary computer network set up for marks data entry worked flawlessly. Also thank you to Sarah Taylor, Executive Officer NZEST for her friendly and efficient management.

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29 December, 1999

- Bit light on waves [**Examiner:** What about A8 (pulse reflection) and B7 (waves & interference)?]
- Not sufficient questions on atomic physics. No questions on dimensional analysis.
- Fairly traditional coverage. No Bohr model, photons, photoelectric effect, atomic line spectra. On the other hand, quite a bit of mechanics (such as statics, straight-line kinematics) which aren't in the new prescription unless you are a lawyer! [**Examiner:** With the statics and kinematics I am unabashedly testing for in-depth knowledge of Form-6 concepts—and will continue to do so.]

4. Length of the paper — was the exam perceived to be too long? too short? just right?

Seven respondents said “**just right**”. Other responses—

- Just right, but one or two bright students commented that the paper was too long.
- Possibly a bit long.
- May have been slightly long (some students gained good marks but failed to attempt B8—or they may not have completed the syllabus)
- Perhaps a bit long given the time needed for thinking in the short answer section.

5. What could be done to improve this Scholarship exam paper?

Several respondents felt strongly that the schol paper should continue to require candidates to perform at a high standard. Here are typical comments:

- The paper should not be “lowered” to allow for the increasing number of low ability candidates now sitting this exam.
- Standard of the paper should be maintained and not allowed to drift down to get higher marks. Students need to come up to this standard.
- I would not like to see the exam standard diluted in order to cater for the lower quality students whose entries were ill-advised and should have been discouraged by their schools.
- I hope you do not drop the standard of next year's exam.
- Rather than cater for the weaker students by being less discriminating and demanding, we could provide a formula sheet which would enable the weak candidates to number crunch some easy marks. [**Examiner:** I intend surveying schools to ask the question: “Should candidates be given a formula sheet?”]

General comments?

- A good paper overall — questions well graded. However some sections should have been examined using practical situations (e.g., B6, B7, and last parts of B5).
- Is the use of log-log paper really necessary given that it is difficult enough at times to get students to use ordinary (linear) paper? At this level, relationships should be plotted in such a way that linear paper produces the desired result.
- The moderators should check the front page constants in future (value for G missing).

NZEST Scholarship Physics 1999

Model Answers and Marking Scheme

Total marks available: 150

Section A: Guided Short-Answer [3 marks each]

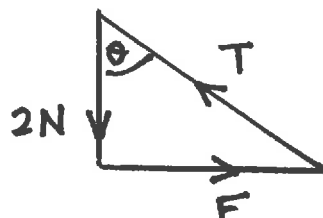
Two marks for reasoning; one mark for correct T/F or multichoice selection.

- 3 1. **False** No. Astronauts report 'weightlessness' because they (and the shuttle) are in free fall towards Earth, hence they experience no reaction force from the walls or 'floor'.
OR No. Gravitational field due to Earth goes as the inverse-square of the distance from the centre of the Earth, $g \propto 1/r^2$. At typical shuttle altitudes (~ 400 km), g is still finite and non-zero. [See 1998 NZEST Physics exam, B.2(d).]
- 3 2. **False** $W = \frac{1}{2}kx^2$, so doubling x will quadruple W . This implies that *additional* work $3W$ will be required (for a total of $4W$).
- 3 3. **False** The time-constant is the time required for the voltage to drop to $1/e \approx 37\%$ of the original value. ($V(t) = V_0 \exp(-t/RC) \rightarrow V_0 \exp(-1) \approx 0.37V_0$ when $t = RC$)
- 3 4. **False** Snapping a bar magnet in two will create two smaller magnets, each with its own north and south poles. (Each will be a dipole magnet.) [Incidentally, isolated magnetic monopoles have never been found.]
- 3 5. **True** As long as the magnetic flux linking the circuit is *changing*, there will be an induced emf. (From Faraday's law, $\varepsilon = -d\Phi/dt$, so it is the *rate of change* of magnetic flux which determines the emf.)
- 3 6. a Complete the vector diagram showing that the forces form a closed triangle.
Since

$$\cos \theta = \frac{\text{adj.}}{\text{hyp.}} = \frac{2}{T},$$

it follows that

$$T = \frac{2}{\cos \theta}.$$



- 3 7. d $I = \sum mr^2 = M \cdot 0 + M \cdot (L/2)^2 + M \cdot L^2 = \frac{5}{4}ML^2$
- 3 8. d Pulse is inverted on reflection from the fixed end. Since the vertical section of the pulse reaches the wall first, it must be the first section to move away from the wall. (Thus the pulse shape is reflected about a horizontal axis *and* about a vertical axis.)

- 3] 9. a The 50-Hz signal generator *drives* the circuit at 50 Hz, so the circuit current must oscillate at 50 Hz. (The current and driver voltage will be out of phase since the driver frequency doesn't match the resonant frequency. There would be zero phase difference and maximum current if the generator frequency were lowered to 16 Hz.)
- 3] 10. d The elapsed time is 12 hours or $12/8 = 1.5$ half-lives, so the count rate will change by factor $(\frac{1}{2})^{1.5}$, giving $N = N_0 \cdot (\frac{1}{2})^{1.5} = (480) \cdot (0.35) = 170$ counts/min.

Section B: Long Questions [15 marks each]

One bonus mark per long question available if student answer shows exceptional insight. Mark subtotals are shown boxed in left margin.

1. Floating, Falling, and Log-log skills

- 3] (a) The bowl remains balanced—nothing happens! ✓ Reason: The lowering of the sugar bowl into the water causes the water level to rise, increasing the pressure acting on the bottom of the dish. This additional pressure is distributed evenly over the entire lower surface, ✓ and does not depend on the location of the floating bowl. ✓ Thus there is no change to the moments acting about the fulcrum at the edge of the bench as the bowl drifts out over the edge. (But what might happen if the bowl were to spring a leak and begin to sink?)
- 3] (b) The heavier object experiences the larger (weight) force, but has a compensating larger inertia ✓✓ (resistance to being accelerated). From Newton's second law ($F = ma$), the acceleration of the heavy object is given by the force per mass: $a = \text{force/mass} = Mg/M = g$, ✓ giving exactly the same acceleration as for the light object.

OR

The larger mass can be thought of as consisting of a number of separate smaller masses which fall together. 'Glueing' these smaller masses together to form the larger mass will not alter their acceleration.

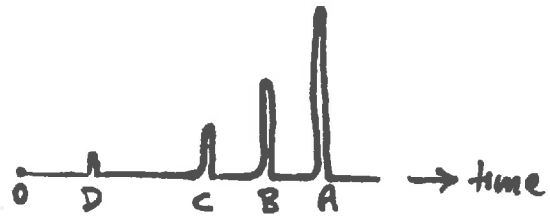
- 3] (c) i. Since the sinkers fall from rest, the distance-time relationship is $h = \frac{1}{2}gt^2$, so the time-of-flight will be given by $t = \sqrt{2h/g}$. ✓ (Refer to third column of the table.)
For crashes to be equally spaced in time, the time-distance relationship would need to be linear ($t \propto h$). ✓ But here we have free-fall under gravity, so $t \propto \sqrt{h}$, a square-root relationship.

Sinker	h (m)	t (s)
A	3.20	0.808
B	2.20	0.670
C	1.20	0.495
D	0.20	0.202

✓

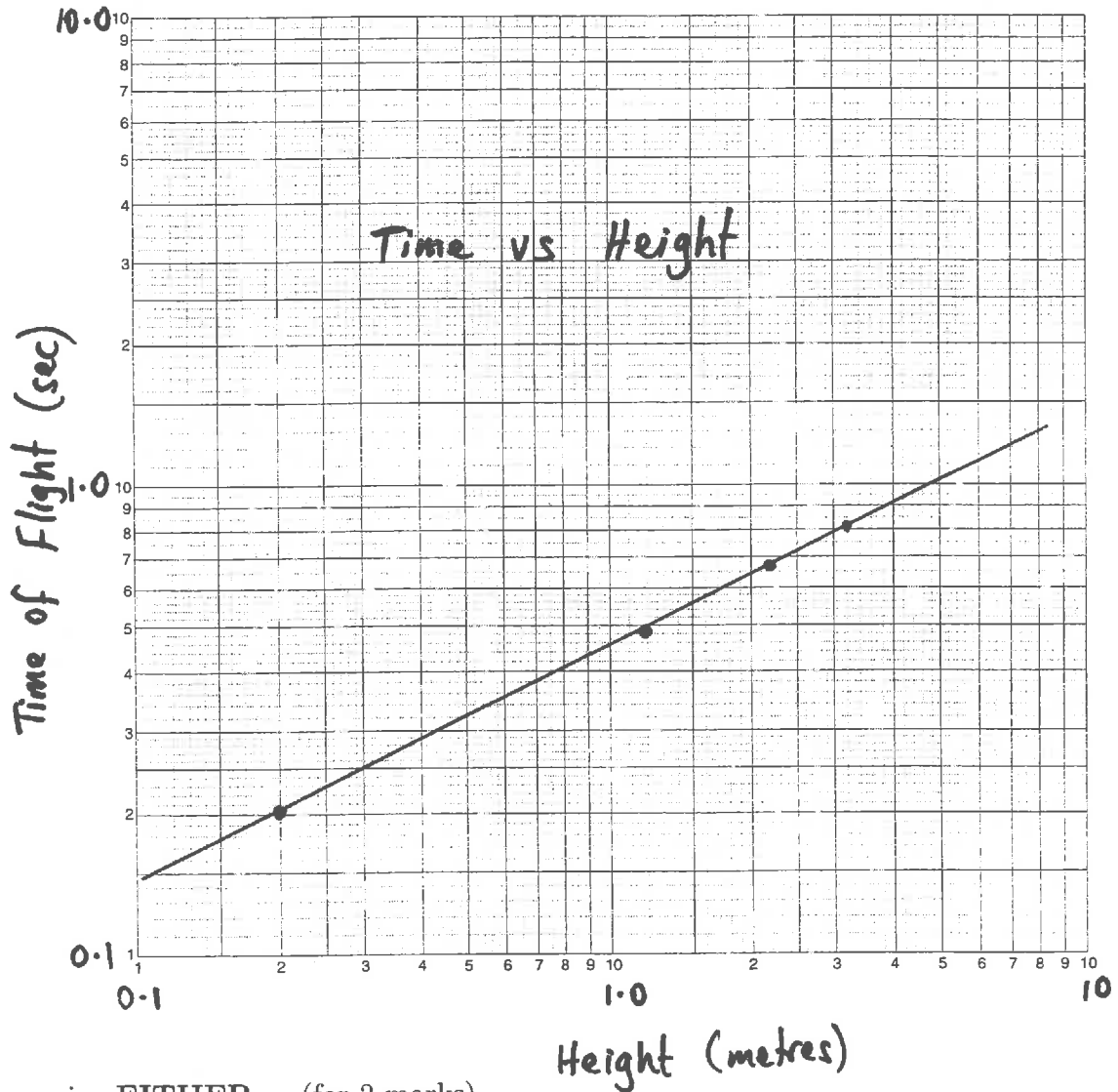
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ii. Expected oscilloscope display:



3

iii. Log-log plot: Time-of-flight vs height



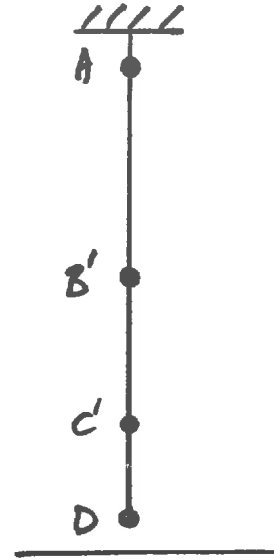
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iv. EITHER (for 2 marks)

The time interval between the D and A crashes is $t_A - t_D = 0.808 - 0.202 = 0.606$ s, so divide this into thirds to give an inter-crash timing of $0.606/3 = 0.202$ s. ✓ Thus (to 1 sig. fig.), crashes occur at 0.2, 0.4, 0.6, 0.8 s. Either read off the revised h_B and h_C heights from the graph, or calculate them from $h = \frac{1}{2}gt^2$. (Refer to table for values. ✓)

Sinker	h' (m)	t' (s)
A	3.20	0.808
B'	1.8	0.606
C'	0.8	0.404
D	0.20	0.202

OR (for 1 mark)
 Show revised B and C sinker locations on a diagram:



2. Plants, Plumb-lines, Candles & Planes

(a) Rotating platform

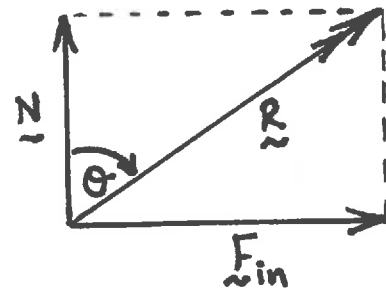
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- i. Vector diagram of soil contact forces. ✓
 The centripetal (inwards) force is given by

$$F_{in} = \frac{mv^2}{r} = m\omega^2 r \quad \checkmark$$

And, from the diagram,

$$\tan \theta = \frac{F_{in}}{mg} = \frac{m\omega^2 r}{mg} = \frac{\omega^2 r}{g} \quad \checkmark$$



1

ii.

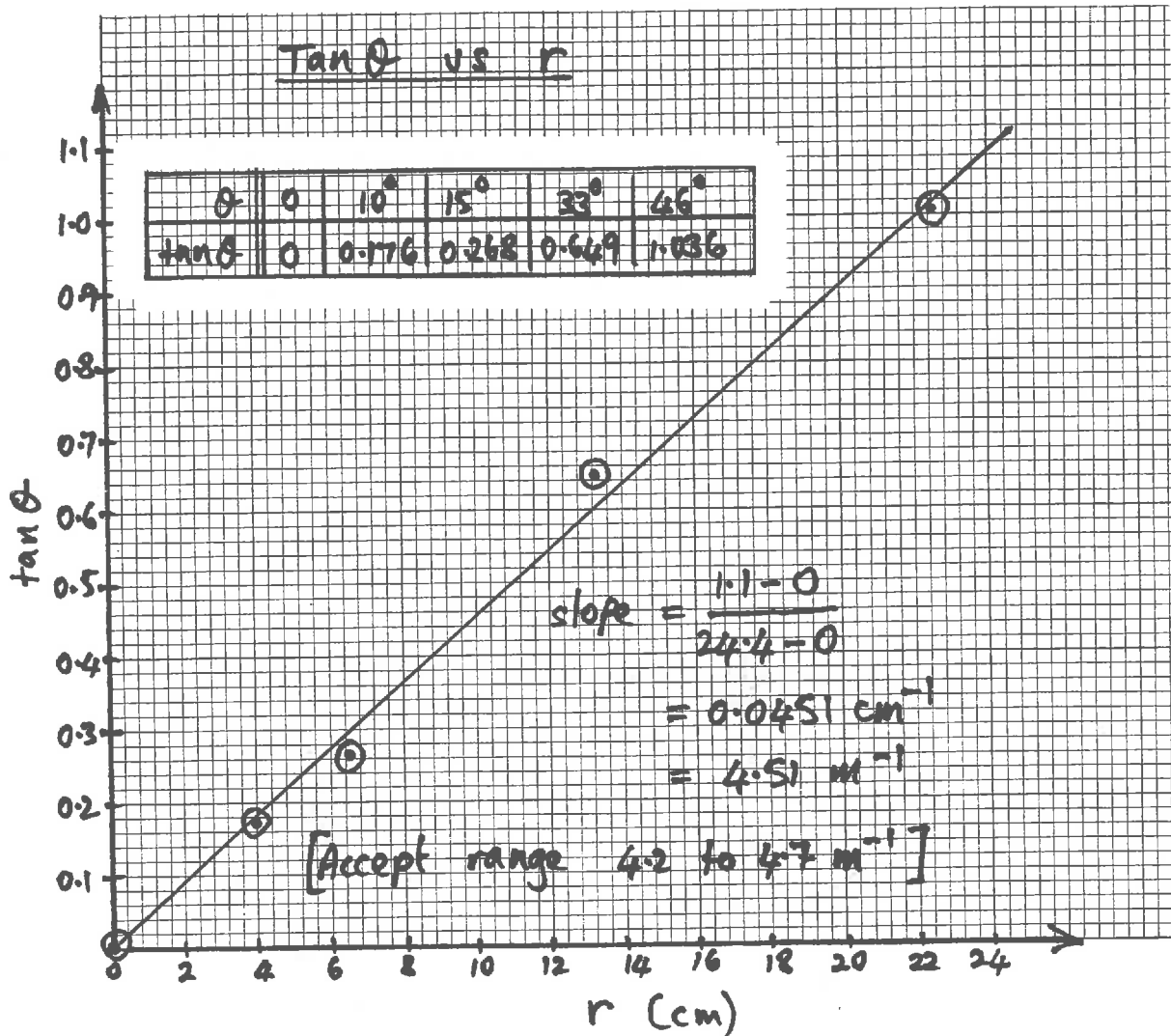
$$65.2 \text{ rev/min} = \left(\frac{65.2 \text{ rev}}{1 \text{ min}} \right) \times \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \times \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 6.83 \text{ rad/s} \quad \checkmark$$

5

iii. Graph of $\tan \theta$ vs r (see next page)—

- Graph hygiene (labels, title) ✓
- Points plotted, best-fit line ✓✓
- Measured value for gradient ✓
- Theoretical gradient prediction:

$$\frac{\omega^2}{g} = \frac{6.83^2}{9.8} = 4.76 \text{ m}^{-1} \quad \checkmark \quad (\text{or } 0.0476 \text{ cm}^{-1})$$



4

iv. Diagram of rotating beaker, plumbline, candle—

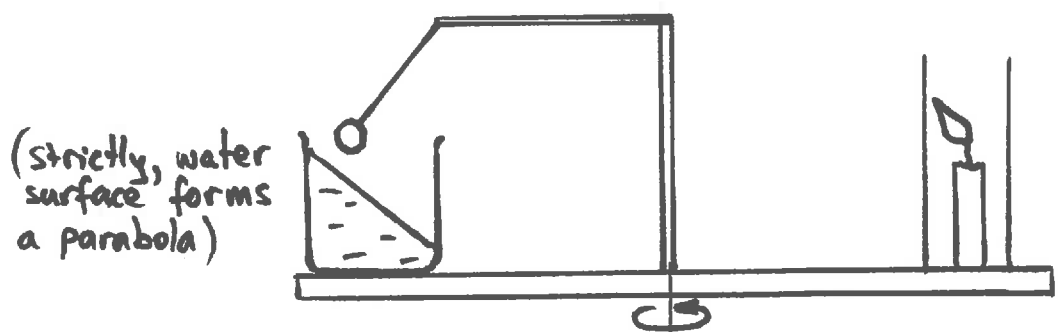


Diagram (1 mark per component) ✓✓✓

Explanation for *one* observation (as below): ✓

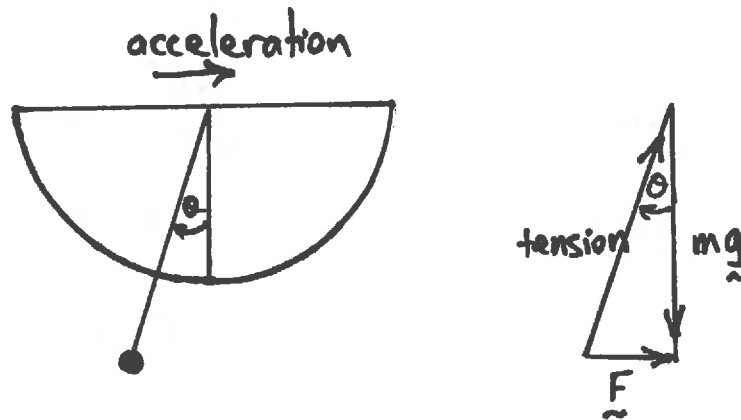
Water surface: Water “piles up” at larger radial distances, producing a pressure gradient because of the increasing depth of water ($P = \rho gh$). This radial pressure increase will cause the water particles to be accelerated (i.e., have their velocity vectors deviated) inward, toward the spin axis.

Plumb-line: The tension in the angled string has an inward component ($T \sin \theta$) required for the bob to move in a circle.

Candle: The flame produces hot, less-dense (buoyant) gases which tend to rise upward in a gravitational field. But the “up” direction is inclined inward relative to the “true” vertical because of the rotation, so the flame also inclines inward. (Or: The less dense gases are displaced to the inside by the denser, colder air molecules which are “pushed” outward as they attempt to move along a tangent path (Newton’s law I).)

- 2 (b) Prior to takeoff, orient the protractor so that its 90° marking faces vertically down, and its plane is parallel to the nose-to-tail axis of the aircraft. Hold the plumb-line so that the string lies along the 90° marking of the protractor, with the string pivoted at the origin of the protractor. When the aircraft accelerates, the plumb-line will incline backwards through angle θ as shown in the diagram. ✓

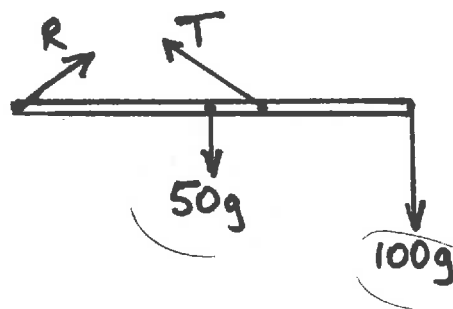
From the forces triangle, $\tan \theta = F/mg$, so $F = ma = mg \tan \theta$, and solving for acceleration we get $a = g \tan \theta$. ✓ (If $\theta = 45^\circ$, then $a = g$.)



3. Statics; SHM (tides); Satellites

(a) Hinged beam

- 1 i. Diagram showing four forces acting on the beam. ✓ (We assign an arbitrary direction for the reaction force R from the hinge.)



- 1 ii. The force R has no turning effect about the hinge, so taking moments about the hinge gives us an equation which depends only on T (and is independent of R). ✓
- 3 iii. At equilibrium, sum of moments about any point is zero. Taking clockwise (CW) moments about the hinge as positive,

$$\sum rF_{\perp} = (7)(50g) + (14)(100g) - (9)(T \sin 37^\circ) = 0 \quad \checkmark$$

Solving for T ,

$$T = \frac{(350 + 1400)g}{9 \sin 37^\circ} = \frac{(1750)(9.8)}{9 \sin 37^\circ} = 3166 \text{ N} = 3.17 \times 10^3 \text{ N} \quad \checkmark\checkmark$$

OR

An alternative method is to generate three equations for the three unknowns T , R , and θ (the angle R makes with the beam)—

Vertical force components add to zero (take upwards positive):

$$R \sin \theta + T \sin 37^\circ - 150g = 0 \quad (1)$$

Horizontal force components add to zero (take rightwards as positive):

$$R \cos \theta - T \cos 37^\circ = 0 \quad (2)$$

Moments about CM of beam add to zero:

$$7R \sin \theta + (7)(100g) - 2T \sin 37^\circ = 0 \quad (3)$$

Or, could take moments about point of attachment of the cable:

$$9R \sin \theta + (5)(100g) - (2)(50g) = 0 \quad (4)$$

Choose any three of these four equations. Solving for the unknowns gives $T = 3.17 \times 10^3 \text{ N}$ (as before), $R = 2.57 \times 10^3 \text{ N}$, and $\theta = -9.8^\circ$ (i.e., R acts 9.8° below the line of the beam).

(b) **Tides diagram**

2

- i. For Raglan, time between consecutive high tides is $17:25 - 4:52 = 12:33 \checkmark\checkmark$ (i.e., 12 hr 33 min).

OR

Time between consecutive low tides is $23:41 - 11:11 = 12:30$ (i.e., 12 hr 30 min).

2

- ii. For Raglan, height variation from first peak to first trough is

$$2.8 - 1.0 = 1.8 \text{ m} \checkmark \implies \text{amplitude} = 1.8/2 = 0.90 \text{ m} \checkmark$$

Other possible answers are

$$(2.7 - 1.0)/2 = 0.85 \text{ m} \quad \text{or} \quad (2.7 - 1.1)/2 = 0.80 \text{ m}$$

(The tide amplitude is only approximately constant.)

2

- iii. Raglan high tide lags Tauranga high tide by

$$\Delta t = 4:42 - 1:59 = 2 \text{ hr } 53 \text{ min} \checkmark$$

Converting this time difference to a phase difference ϕ (in degrees),

$$\phi = \frac{\Delta t}{T} \cdot 360^\circ = \left(\frac{2 \frac{53}{60}}{12 \frac{33}{60}} \right) \cdot 360^\circ = \left(\frac{2.88}{12.55} \right) \cdot 360^\circ = 82.6^\circ$$

i.e., Raglan tide lags Tauranga tide by $83^\circ \checkmark$

- 2 iv. The angular frequency for the Raglan tide is $\omega = 2\pi/T = 2\pi/12.55 = 0.50$ rad/hr. ✓ Taking the Raglan tide amplitude as $A = 0.90$ m, the maximum rate of change of height is given by $v_{\max} = \omega A = (0.50)(0.90) = 0.45$ m/hr. ✓

2 (c) **Science fiction**—

“Geostationary” means stationary with respect to an Earth observer. This can only be achieved if two conditions are satisfied: (1) The satellite must have an orbit period of 24 hr; and (2) the satellite must travel above the equator. An orbit trajectory which is stationary over London is therefore impossible. ✓✓

4. Gravity and Electricity

(a) Substantial persons

- 1 i. Force—

$$F_{\text{grav}} = \frac{Gm_A m_B}{r^2} = \frac{(6.67 \times 10^{-11})(100)(100)}{1^2} = 6.67 \times 10^{-7} \text{ N } \checkmark$$

- 2 ii. Acceleration—

$$a = \frac{F}{m_A} = \frac{6.67 \times 10^{-11}}{100} = 6.67 \times 10^{-9} \text{ m/s}^2 \checkmark$$

This acceleration would increase as the persons approach each other since the gravitational force would increase. ✓

(b) Charged persons

- 1 i. Force—

$$F_{\text{elec}} = \frac{kq_A q_B}{r^2} = \frac{(8.99 \times 10^9)(2 \times 10^{-6})(-2 \times 10^{-6})}{1^2} = -3.60 \times 10^{-2} \text{ N}$$

i.e., an attractive force of 3.60×10^{-2} N ✓

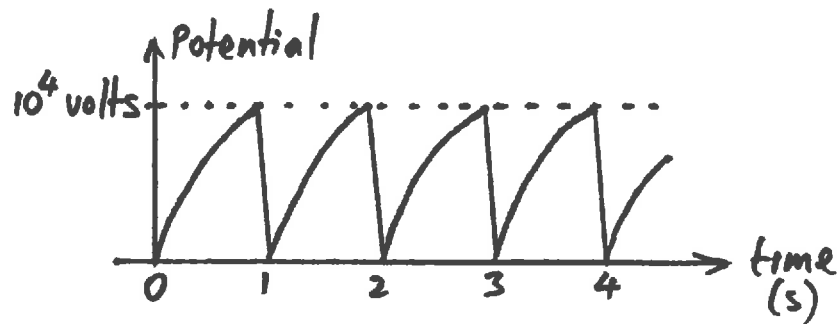
- 2 ii. Force ratio—

$$\frac{F_{\text{elec}}}{F_{\text{grav}}} = \frac{3.60 \times 10^{-2}}{6.67 \times 10^{-7}} = 5.39 \times 10^5 \checkmark$$

The electrostatic force is *much* larger (about 50,000 times!) than the gravitational force. ✓

- 2 iii. Masses of planets and stars are huge; ✓ net charge carried by these planets and stars is tiny, presumably because separated charges will tend to move together and neutralize. ✓ There is no obvious way of “charging” a planet (solar wind perhaps?); and even if you could, the charge-holding capacity of a planet is rather small (e.g., for a spherical Earth, $C = 4\pi\epsilon_0 R_E \approx 700 \mu\text{F}$).

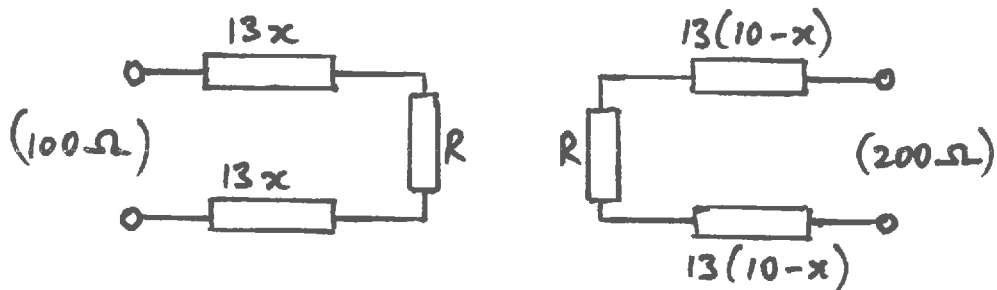
(c) Van de Graaf generator



- 3 i. The charging portion will be *linear* if the rate of charge delivery to the dome is constant ($V = Q/C = \frac{1}{C} \int I dt$); however, as charge accumulates on the dome, the belt will experience a larger repulsive force and may slip, in which case the charging rate would decrease with time, leading to a *curved* charging graph.
 Labelled graph ✓✓
 Justified choice of linear or curved ✓
- 2 ii. Taking the dome as a complete sphere (i.e., ignore the hole for the belt!), its capacitance is $C = 4\pi\epsilon_0 R = 4\pi(8.85 \times 10^{-12})(0.20) = 2.22 \times 10^{-11} \mu\text{F}$. Using $Q = CV$ with $V = V_{\text{max}} = 10^4 \text{ V}$, the maximum charge will be $Q_{\text{max}} = (2.22 \times 10^{-11})(10^4) = 2.22 \times 10^{-7} = 0.222 \mu\text{C}$. ✓✓
- 1 iii. Assuming the current is constant for the duration of the spark, $I = \Delta Q / \Delta t = 2.22 \times 10^{-7} / 0.05 = 4.45 \times 10^{-6} \text{ A}$. ✓
 OR Using $Q_{\text{max}} = 0.6 \mu\text{C}$, we get $I = 1.20 \times 10^{-5} \text{ A}$.
- 1 iv. Electrostatic energy is $E = \frac{1}{2}CV^2 = \frac{1}{2}(2.22 \times 10^{-11})(10^4)^2 = 1.11 \times 10^{-3} \text{ J}$. ✓
 OR $E = \frac{1}{2}Q^2/C = \frac{1}{2}(2.22 \times 10^{-7})^2 / (2.22 \times 10^{-11}) = 1.11 \times 10^{-3} \text{ J}$.
 OR $E = \frac{1}{2}QV = \frac{1}{2}(2.22 \times 10^{-7})(10^4) = 1.11 \times 10^{-3} \text{ J}$.

5. Resistors, Capacitors, and AC Circuits

- 5 (a) Failed cable
 i. West- and east-view circuit diagrams (below): ✓✓



ii. The two circuits give two equations in two unknowns:

$$100 = 26x + R \quad (5)$$

$$200 = 26(10 - x) + R \quad (6)$$

$$(6) - (5): \quad 100 = 260 - 52x$$

$$x = \frac{160}{52} = 3.08 \text{ km} \quad \checkmark$$

$$\text{Substituting } x \text{ in (5):} \quad R = 100 - 26x = 100 - 26 \left(\frac{160}{52} \right) = 20.0 \Omega \quad \checkmark$$

2 (b) Capacitor bridge

We can view the capacitor bridge as consisting of three capacitances in parallel:

$$C_{ab} = C_{\text{upper}} + C_{\text{middle}} + C_{\text{lower}}, \text{ where } C_{\text{upper}} = C_{\text{lower}} = C_0/2.$$

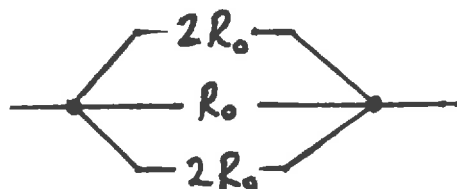
$$\text{Thus } C_{ab} = C_0/2 + C_0 + C_0/2 = 2C_0. \quad \checkmark \checkmark$$

2 (c) Resistor bridge

Treating the resistor bridge as three resistances in parallel,

$$\frac{1}{R_{ab}} = \frac{1}{2R_0} + \frac{1}{R_0} + \frac{1}{2R_0} = \frac{4}{2R_0}$$

$$R_{ab} = \frac{R_0}{2} \quad \checkmark \checkmark$$



(d) RLC circuit

1 i. $\omega_0 = 1/\sqrt{LC} = 1/\sqrt{(200 \times 10^{-3})(0.050 \times 10^{-6})} = 1.00 \times 10^4 \text{ rad/s} \quad \checkmark$

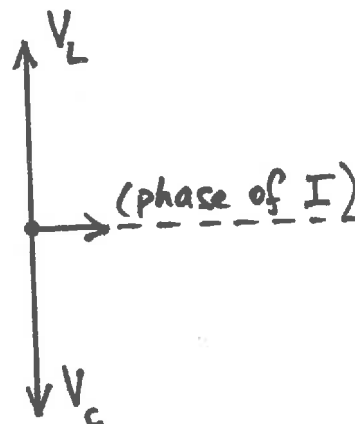
3 ii. At resonance, the impedance of the circuit is $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2} = R$, since the two reactance terms cancel. The rms current will therefore be $I_{\text{rms}} = V_{\text{rms}}/Z = 150/50.0 = 3.00 \text{ A}$. The rms voltage across each element will be—

$$V_{ab} = IR = (3.00)(50.0) = 150 \text{ V} \quad \checkmark$$

$$V_{bc} = I\omega L = (3.00)(1.00 \times 10^4)(200 \times 10^{-3}) = 6.00 \times 10^3 \text{ V} \quad \checkmark$$

$$V_{cd} = I/(\omega C) = (3.00)/[(1.00 \times 10^4)(0.050 \times 10^{-6})] = 6.00 \times 10^3 \text{ V} \quad \checkmark$$

2 iii. $V_{bd} = 0 \quad \checkmark$ because the V_L and V_C phasors are equal and opposite (in exact antiphase), so cancel. Refer to the diagram. \checkmark



6. Magnetic Force, Magnetic Flux, and Induced emf

2 (a) Magnetic flux is the product of magnetic field strength and the normal component of the area over which it acts. ✓ $\phi_B = BA \cos \theta$ where θ is the angle between \vec{B} and the area normal. ✓ Units: tesla·metre² (Tm²) or weber (Wb).

3 (b) To demonstrate emf generation, first connect the two ends of the wire to a (centre-zero) voltmeter, then vary the flux linkage. Methods for doing this include:

(1) **translating** the wire relative to the magnetic field ✓ (one emf polarity on entry; opposite polarity on exit from field; no emf if no relative movement between conductor and field);

(2) **rotating** a small loop of the wire within the field (about an axis perpendicular to the field); ✓

(3) **collapsing** (or increasing) the **area** of the loop within the field by pulling (pushing) on both ends of the wire. ✓

(c) Pulled loop

1 i. $\phi_B = BA = BLx$ ✓ = (2.0)(0.04)x = 0.08x

1 ii. $\phi_B = BLb = (2.0)(0.04)(0.10) = 8.0 \times 10^{-3}$ Wb ✓

3 iii. $\varepsilon = d\phi_B/dt = BLv = (2.0)(0.04)(1.0) = 0.08$ V ✓

The emf polarity is positive at the top of the leading edge of the loop, so induced (conventional) current in the loop will be anticlockwise ✓ (inducing a secondary magnetic field directed out of the page). Induced current is $I = \varepsilon/R = 0.08/1.6 = 0.05$ A = 50 mA. ✓

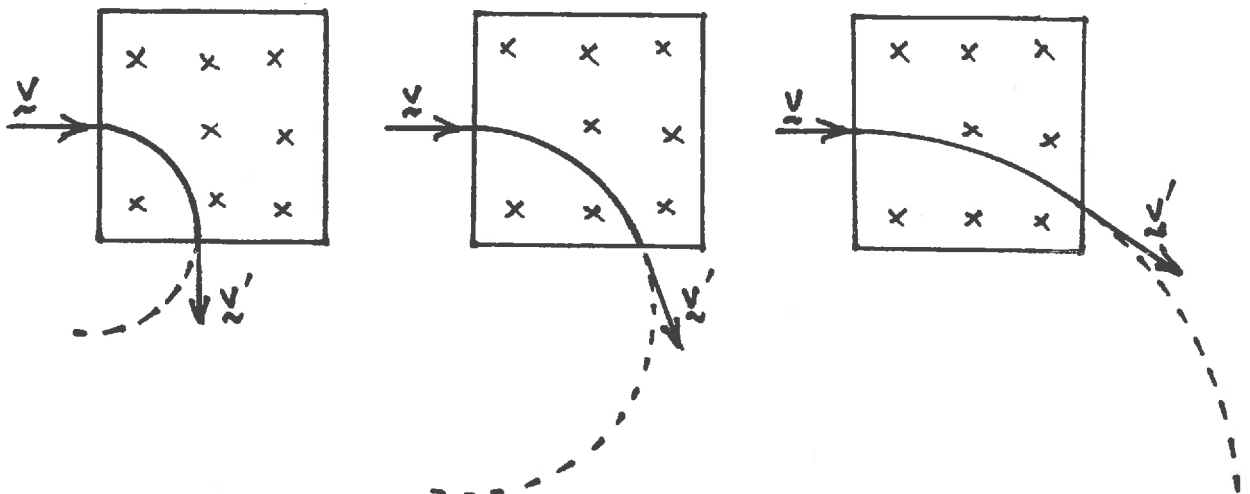
1 iv. Zero, since there is no rate change of flux ($\phi_B = \text{constant}$, so $d\phi_B/dt = 0$). ✓
OR Each end of the loop will generate equal but opposing emfs, so there is no net emf generated.

(d) Electron in magnetic field

1 i. Magnetic force will act *down* the page. ✓

2 ii. $F = Bev = (2.0)(1.60 \times 10^{-19})(3.0 \times 10^{+6}) = 9.60 \times 10^{-13}$ N ✓✓

1 iii. Electron trajectories—



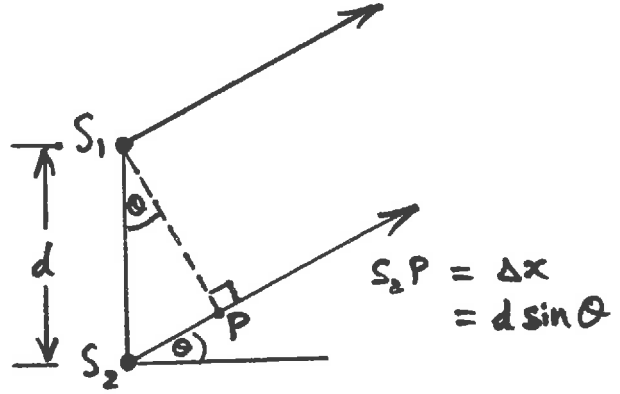
While within the field, the electron will travel along a *circular arc*; when it exits the field, the trajectory will be a straight line. ✓ The diagram (previous page) shows three possible trajectories.

7. Waves and Interference

(a) Young's double-slit

2

- i. From the right-triangle, $d \sin \theta$ is the extra distance travelled by waves from S_2 . ✓ In order to obtain constructive interference at the distant screen, the waves from both sources must arrive in phase. This can only happen if the extra distance travelled by the S_2 waves is a whole number of wavelengths. ✓



1

- ii. $\sin \theta = m\lambda/d = (1)(\lambda)/(100\lambda) = 0.01 \implies \theta = 0.01 \text{ rad}$ ✓

2

- iii. $\Delta x = d \tan \theta = (0.50)(\tan 0.01) = 5 \times 10^{-3} \text{ m}$ ✓✓ = 5 mm

OR θ is small, so can calculate Δx from the arc-distance formula $s = r\theta$, giving $\Delta x = d \cdot \theta = 5 \times 10^{-3} \text{ m}$.

1

(b) Diffraction grating

$$d = \frac{1 \text{ mm}}{600} = 1.67 \times 10^{-3} \text{ mm} \checkmark = 1.67 \times 10^{-6} \text{ m}$$

(c) Reflection grating

2

- i. For zeroth order, $m = 0$, so $d(\cos \theta_{\text{in}} - \cos \theta_{\text{out}}) = 0$.
But $d \neq 0 \implies \cos \theta_{\text{in}} = \cos \theta_{\text{out}} \implies \theta_{\text{in}} = \theta_{\text{out}}$ ✓✓

7

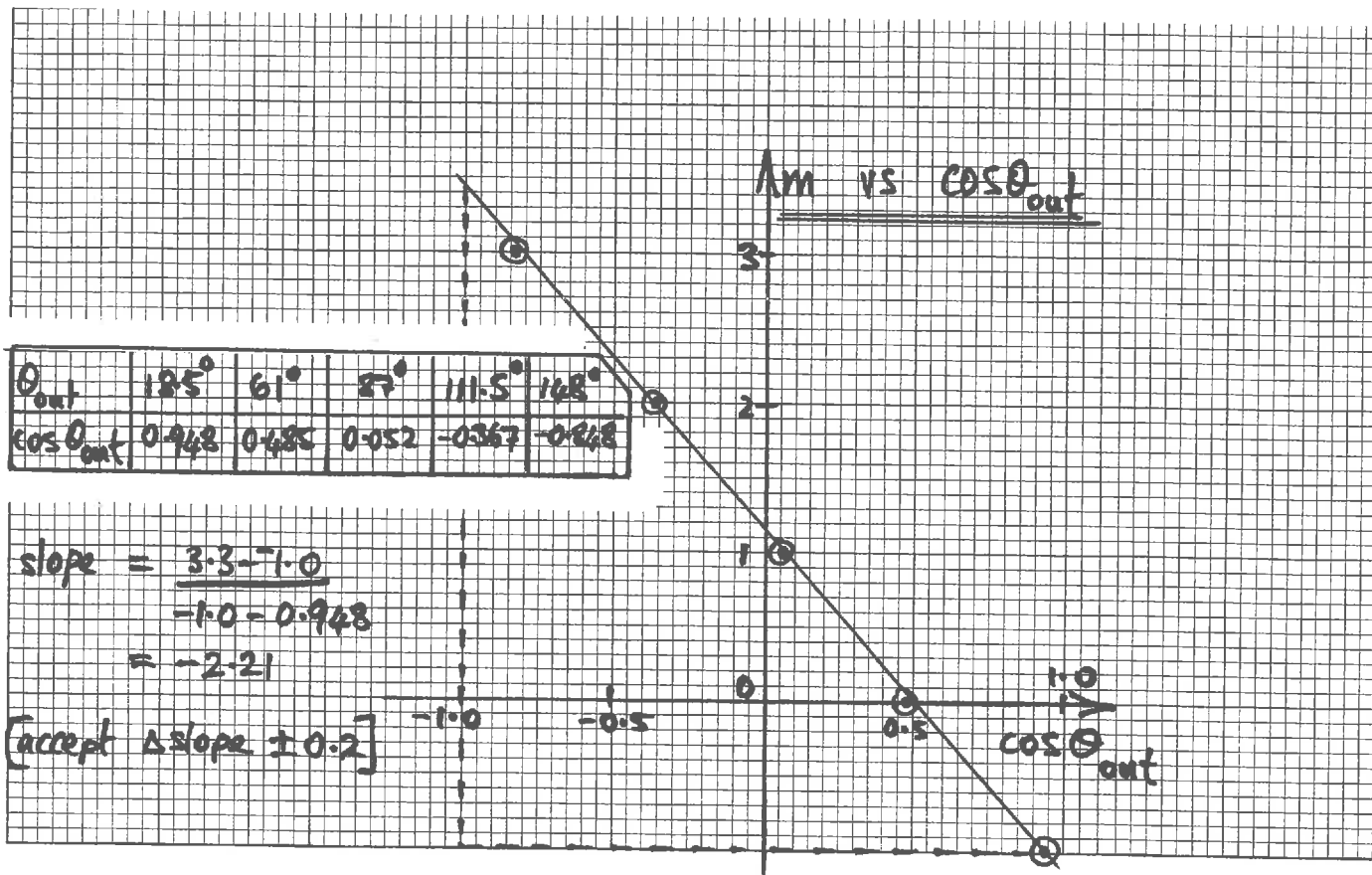
- ii. Substituting $\cos \theta_{\text{in}} = \cos 60^\circ = \frac{1}{2}$ in the diffraction equation and solving for m ,

$$\begin{aligned} m &= \frac{d}{\lambda} \cos \theta_{\text{in}} - \frac{d}{\lambda} \cos \theta_{\text{out}} \\ &= \frac{d}{2\lambda} - \frac{d}{\lambda} \cos \theta_{\text{out}} \\ &= (\text{intercept}) + (\text{slope}) \cdot \cos \theta_{\text{out}} \end{aligned}$$

shows that a graph of m (y -axis) vs $\cos \theta_{\text{out}}$ (x -axis) should give a straight line of slope $= -d/\lambda$ and y -intercept $= d/(2\lambda)$ (thus we could use *either* the slope *or* the intercept to determine d).

From the graph, slope $= -2.21$, so $d = 2.21\lambda = (2.21)(633 \times 10^{-9}) = 1.40 \mu\text{m}$.
(Accept tolerance of ± 0.20 in slope, and $\pm 0.10 \mu\text{m}$ in d .)

Graph (next page): labelled axes, title, values, line of best fit ✓✓✓✓
Slope: ✓✓ Value for d : ✓



8. Nuclear & Atomic Physics; Order-of-Magnitude Estimation

1 (a) Aldiss "science"

The Aldiss formula suggests that *more* energy would be released per unit mass than previously! ✓ Therefore less matter would be required to produce one joule of energy (not more, as claimed). There is also a dimensions problem: the right-hand side has to have units of joules = kgm^2/s^2 , but this cannot be the case if c^2 is replaced by $c^{2.7713}$.

3 (b) $E = mc^2 = (1.66054 \times 10^{-27})(2.997925 \times 10^8)^2 = 1.492419 \times 10^{-10} \text{ J}$ ✓

But $1 \text{ eV} = 1.602177 \times 10^{-19} \text{ J}$, ✓ so

$E = 1.492419 \times 10^{-10} / 1.602177 \times 10^{-19} = 9.314947 \times 10^8 \text{ eV} = 931.5 \text{ MeV}$ ✓

(c) Sun fusion

1 i. X is a positron ✓ (0_1e)

3 ii. Energy release—

$$\begin{aligned} \text{total mass of LHS (4 protons)} &= 4.029104 \text{ u} \\ \text{mass of helium nucleus} &= 4.001510 \text{ u} \\ \text{mass of 2X} &= 0.001098 \text{ u} \\ \text{total mass of RHS} &= 4.002608 \text{ u} \\ \text{mass deficit: LHS} - \text{RHS} &= 0.026496 \text{ u} \checkmark\checkmark \end{aligned}$$

This “missing” mass has been released as energy:

$$E = (0.026496 \text{ u})(931.5 \text{ MeV/u}) = 24.68 \text{ MeV} \checkmark$$

- 2 (d) Laser light is completely coherent, highly directional (radiates plane waves), monochromatic (single wavelength), and very intense. \checkmark Light from a domestic incandescent bulb is almost completely incoherent, non-directional (radiates spherical waves from a “point” source), produces a broad spectrum across the infrared and visible range, and is very much less intense than laser light. \checkmark (A 60-W laser is probably powerful enough to punch holes in steel! A hand-held laser pointer is typically rated at ~ 1 mW.)

- 5 (e) **Car tyre wear:** There is no “correct” answer. We are looking for a reasonable chain of logic. Here is one approach:

Diameter of tyre, $d \approx 0.5 \text{ m}$

Thickness of tread, $t \approx 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$

Expected life distance, $D \approx 40,000 \text{ km} = 4 \times 10^7 \text{ m}$

Circumference of tyre, $C = \pi d \approx 1.5 \text{ m}$

Tyre revs for 40,000 km, $n = D/C = 4 \times 10^7 / 1.5 \approx 3 \times 10^7$

So wear per revolution, $\Delta t = t/n = 5 \times 10^{-3} / 3 \times 10^7 \approx 2 \times 10^{-10} \text{ m}$
