AS91524: Demonstrate understanding of electrical systems Level 3 Credits 6

Achievement	Achievement with Merit	Achievement with Excellence
Demonstrate understanding of electrical systems.	Demonstrate in-depth understanding of electrical systems.	Demonstrate comprehensive understanding of electrical systems.

Resistors in DC Circuits

Internal resistance; simple application of Kirchhoff's Laws.

Capacitors in DC Circuits

Parallel plate capacitor; capacitance; dielectrics; series and parallel capacitors; charge/time, voltage/time and current/time graphs for a capacitor; time constant; energy stored in a capacitor.

Inductors in DC Circuits

Magnetic flux; magnetic flux density; Faraday's Law; Lenz's Law; the inductor; voltage/time and current/time graphs for an inductor; time constant; self inductance; energy stored in an inductor; the transformer.

AC Circuits

The comparison of the energy dissipation in a resistor carrying direct current and alternating current; peak and rms voltage and current; voltage and current and their phase relationship in LR and CR series circuits; phasor diagrams; reactance and impedance and their frequency dependence in a series circuit; resonance in LCR circuits.

Relationships:

$E = \frac{1}{2}QV$ $\quad Q = CV$ $\quad C = \frac{\varepsilon_o \varepsilon_r A}{d}$ $\quad C_T = C_1 + C_2 + \ldots$ $\quad \tau = RC$

$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$ $\quad \phi = BA$ $\quad \varepsilon = -L\frac{\Delta I}{\Delta t}$ $\quad \varepsilon = -\frac{\Delta \phi}{\Delta t}$

$\frac{N_p}{N_s} = \frac{V_p}{V_s}$ $\quad E = \frac{1}{2}LI^2$ $\quad \tau = \frac{L}{R}$

$I = I_{MAX} \sin \omega t$ $\quad V = V_{MAX} \sin \omega t$ $\quad I_{MAX} = \sqrt{2} I_{rms}$

$V_{MAX} = \sqrt{2} V_{rms}$ $\quad X_C = \frac{1}{\omega C}$

$X_L = \omega L$ $\quad V = IZ$ $\quad \omega = 2\pi f$ $\quad f_0 = \frac{1}{2\pi\sqrt{LC}}$

Level 2 Electricity

All of the work from level 2 is assumed knowledge here:

$$I = \frac{q}{t} \qquad V = \frac{\Delta E}{q}$$

$$V = IR \qquad P = IV$$

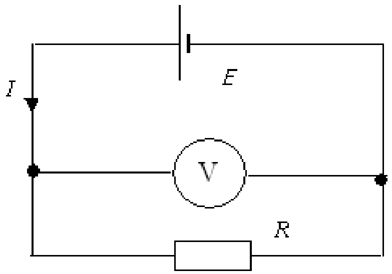
$$R_T = R_1 + R_2 + \ldots \qquad \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

(All of the above you should know from Level 2 – and if you don't you need to)

Internal Resistance

The internal resistance, r, of a power source affects circuits.

The internal resistance of the power source should be included in any voltage or current calculations (At Level 3 this will be requested if required).



E = V + Ir (Note: This equation is not given to you at Level 3)

where **E**= EMF, **V** = Terminal p.d. and **Ir** is the lost volts.

When current flows round a circuit energy is transformed in both the external resistor but also in the cell itself.

It is possible to analyse voltage/current graphs for a circuit to determine the internal resistance of a power source. The effect of r on the circuit depends on the external load resistance, R, in the circuit.

- When R is very large, I is very small so r has no effect and V = E
- When R is very small, I is relatively large so r has effect and V < E
- When R = r, this is impedance matching and the maximum power is gained from the voltage source

Kirchoffs law 1

Calculation of voltages and currents in one- and two-loop circuits, which may include DC voltage sources and resistors must be done by using Kirchhoff's laws:

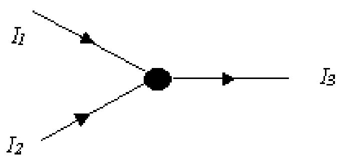
The sum of the potential differences in any loop is zero

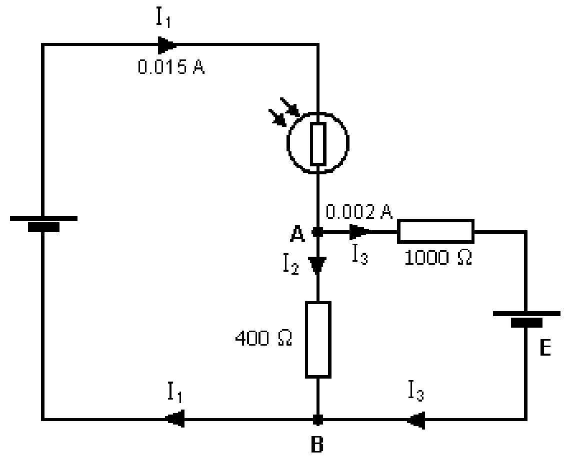
$\Sigma V = 0$ (Note: This equation is not given to you at Level 3)

Kirchoffs law 2

The sum of the currents entering a junction = The sum of the currents leaving a junction (There is no build-up of charge at a junction)

$\Sigma I = 0$ (Note: This equation is not given to you at Level 3)





Applying Kirchoff's first law to junction A:
Current in the 400 Ω resistor = 0.015 - 0.002 = 0.013 A

Potential difference across 400 Ω resistor = 0.013 x 400 = 5.2 V
This is the potential difference between A and B via the 400 Ω resistor but it is also the potential difference across the right hand branch of the circuit via the cell of E.M.F, E.

The potential difference across the 1000 Ω resistor is 0.002 x1000 = 2 V

Applying Kirchhoff's second law to the right hand branch and considering an anticlockwise direction from the cell:
E.M.F of the cell (E) = -0.002 x 1000 + 5.2
= -2 + 5.2 = 3.2 V

(A solution using simultaneous equations will only be required at Scholarship level)

Capacitors

A **capacitor** in a DC circuit stores charge – charge gathers on one plate and is repelled from the other plate – charges never move between the plates inside a capacitor.

$$Q = CV$$

Where Q = charge(C), C = capacitance (F) and V = Voltage (V)

Understand the parallel plate capacitor by using the formula below:

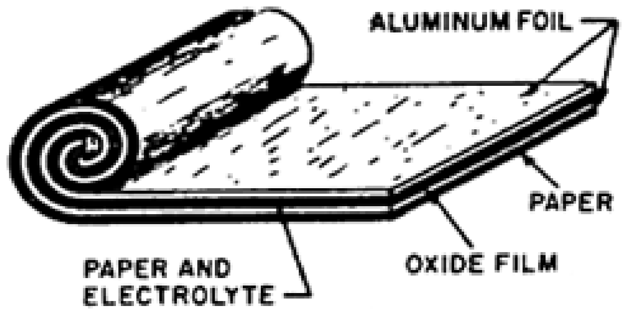
$$C = \frac{\varepsilon_o \varepsilon_r A}{d}$$

Where C = Capacitance (F), ε_0 = absolute permittivity (Fm^{-1}), ε_r = dielectric constant (A.K.A relative permittivity), A = overlap area of plates (m^2), d = distance between plates

Dielectric Constants

MATERIAL	CONSTANT
Vacuum	1.0000
Air	1.0006
Paraffin paper	2.5 - 3.5
Transformer oil	4
Glass	5 - 10
Mica	3 - 6
Rubber	2.5 - 35
Wood	2.5 - 8
Porcelain	6
Glycerine (15 C)	56
Petroleum	2
Pure water	81

To make a capacitor smaller, most capacitors are rolled up.



The formula still applies. Most capacitors are so small that they are measured in μF = 10^{-6} F , nF = 10^{-9} F or even pF = 10^{-12} F

Capacitor Networks

The total capacitance for series and parallel capacitor combinations can be calculated:

$C_T = C_1 + C_2 + \dots$ for capacitors in parallel

$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$ for capacitors in series

(The opposite method to adding resistors)

The energy stored in a capacitor can be calculated by:

$$E = \tfrac{1}{2}QV$$

Where E = energy (J), Q = charge(C) and V = Voltage (V)

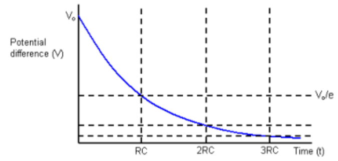
Note: A capacitor in a D.C. circuit is only 50% efficient

(Energy supplied, E = QV; E stored, E = ½QV)

Charging and discharging

Capacitors can charge up (gain charge) and discharge (lose charge). A capacitor is regarded to be fully charged or fully discharged after 5 x **τ**. This **charging and discharging** is non-linear.

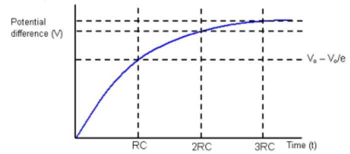
Discharging a capacitor



$$\tau = RC$$

This is the **time constant** for the circuit when V = 37% of V_o. The bigger the value of RC the slower the rate at which the capacitor discharges.

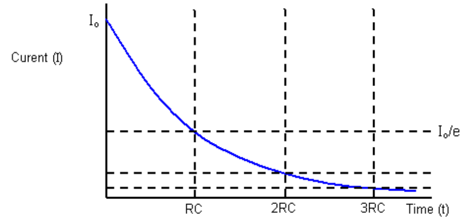
Charging a capacitor



$$\tau = RC$$

This is the **time constant** for the circuit when V reaches 63% of V_o.

In both cases the current looks like this:



Magnetic fields

Magnetic fields can be described in terms of magnetic flux, ∅ (Wb), and magnetic field strength, B (T).

They are linked by the equation

$$\phi = BA$$

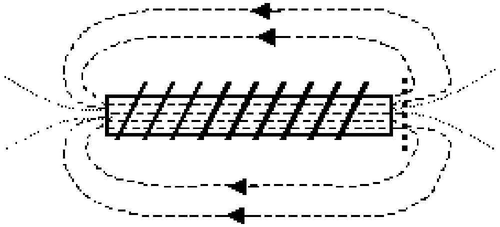
Where A = area (m^2)

Magnetic fields are find outside and inside magnets (and are strongest at the poles).

A solenoid also behaves like a magnet since a moving current produces a magnetic field

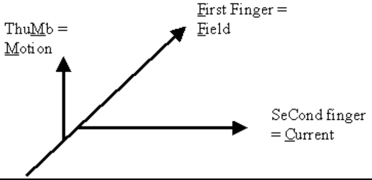
Magnetic fields can be produced by:

- single wire
- coil (solenoid)

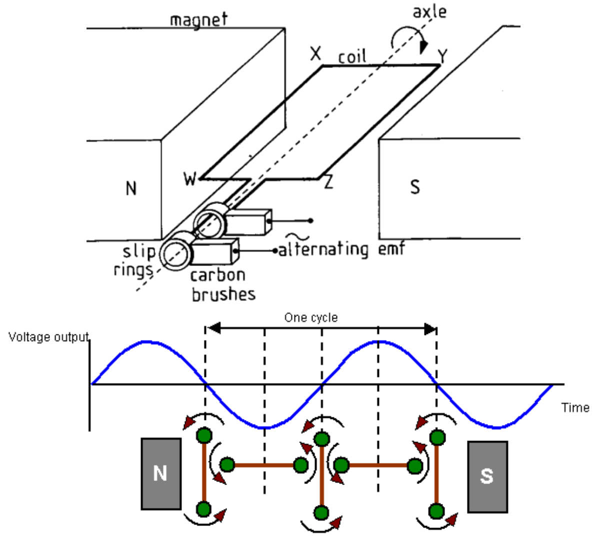


Electromagnetic induction

When 2 of **M**ovement, Magnetic **F**ield and **C**urrent exist at an angle to each other (90° is the optimum angle) the third is induced:



Electromagnetic induction is used in an AC Generator:



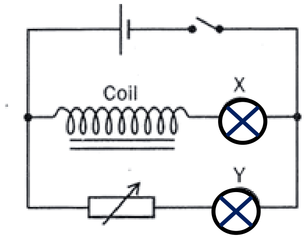
The induced Voltage can be calculated by using Faraday's law:

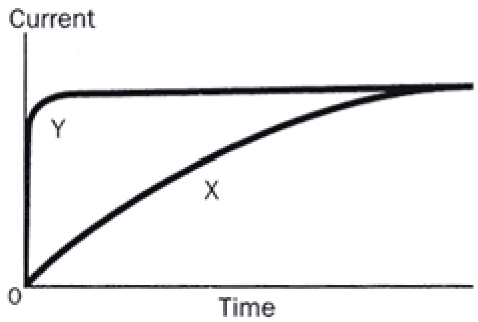
$$\varepsilon = -\frac{\Delta\phi}{\Delta t}$$

Where ε = EMF(V), L= inductance (H), ΔI/ Δt = rate of change of current (As^{-1})

Inductors

Inductors can produce magnetic fields when current is **changing** within the coil. They oppose **any change** in current (either an increase or a decrease). This **opposition** is non-linear (just like capacitors)





Y represents what would happen to the current without an inductor, X with an inductor.
The **time constant** for this circuit (when I reaches 63% of maxiumum I) is given by:

$$\tau = \frac{L}{R}$$

Where τ = time constant(s), R = resistance (Ω) and L =inductance (H)

An inductor is regarded to have a stable magnetic field or no magnetic field after 5 x τ (although theoretically this never happens)

Inductors store energy as magnetic fields, given by:

$$E = \frac{1}{2} L I^2$$

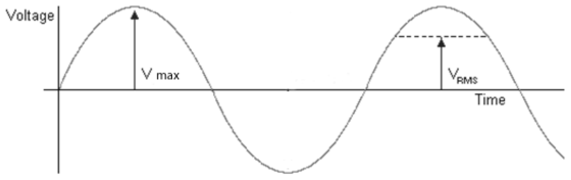
The curve above can also be explained using Lenz's law and the creation of a back EMF

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

Where ε = EMF(V), L= inductance (H), ΔI/ Δt = rate of change of current (As^{-1})

Electricity Generation

Electricity can be generated by a coil rotating with a constant angular velocity in a uniform magnetic field – This is what happens in nearly all power stations worldwide.



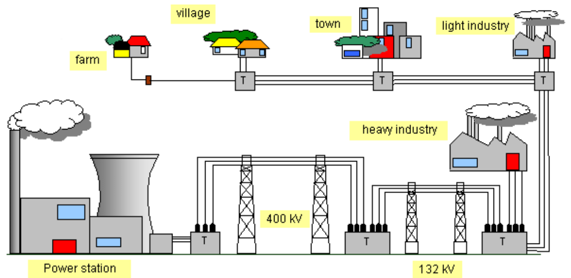
$$V = V_{MAX} \sin \omega t$$

$$I = I_{MAX} \sin \omega t$$

These are the consequences of a coil rotating with constant angular velocity where ω is the angular frequency of the coil (s^{-1}) and t is the instantaneous time (s).

I_{MAX} and V_{MAX} are the maximum values of the current produced when the movement of the coils is 90° to the magnetic field.

Domestic Mains electricity in New Zealand is 240 V 50Hz. However, due to energy losses at medium to high currents, electricity is converted to 330,000 Volts for transmission around NZ.



Transformers use mutual inductance

$$\varepsilon = -M \frac{\Delta I}{\Delta t}$$

Where ε = EMF (V), M= mutual inductance (H), $\Delta I / \Delta t$ = rate of change of current (As^{-1}).

Two inductors linked by a common magnetic field are more commonly called a transformer.

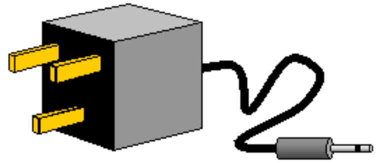
$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

Where N_p = number of turns on primary, N_s = number of turns on secondary, V_p = voltage supplied to primary and V_s = voltage induced on secondary.

Transformers can step up or step down Voltages with high efficiency. Effective inductors require **changing** currents so A.C. electricity is used for distribution of mains electricity (except for across Cooks Strait which is no longer Level 3)

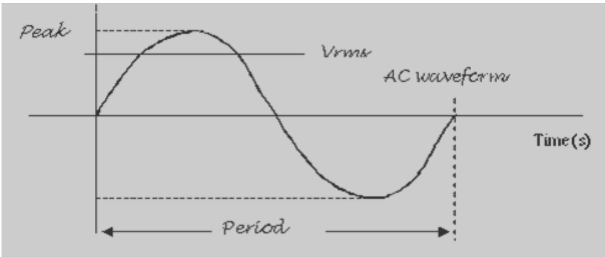
If the number of turns on the primary is greater than that on the secondary then the voltage on the secondary will be less than that on the primary If there are more turns on the secondary the output voltage will be larger.

Computers step down the Voltage (therefore stepping up the current, since the Power remains the same).



Using AC

The energy dissipation of a resistor carrying DC and AC is identical as is bulb brightness (power) using AC and DC providing the root mean square(rms) value of the A.C. matches the D.C. value.



RMS values are a method of averaging sine waves that would otherwise be calculated as zero.

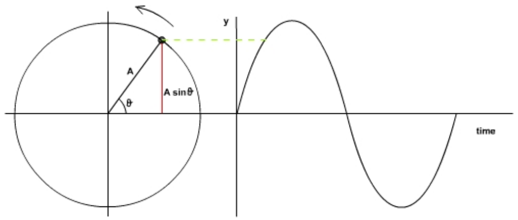
$$I_{MAX} = \sqrt{2}\, I_{rms}$$

$$V_{MAX} = \sqrt{2}\, V_{rms}$$

Mains electricity actually varies between + 340 V and – 340 V with 240 V being the rms value.

A.C. can be converted to DC by the use of bridge diode circuits by full wave rectification.

A.C. electricity can be described by drawing a sine wave with time period 0.02 s – it is best drawn using phasors.

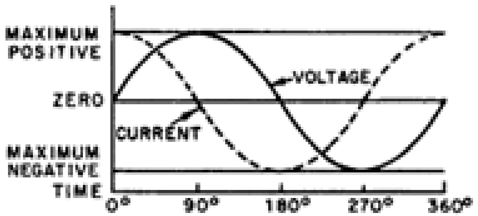


Reactance

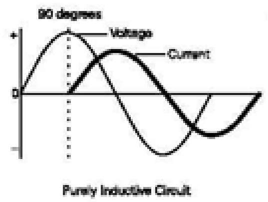
The current and voltage for a resistor in an AC circuit are in phase (resistors are passive in nature).

However:

Current **leads** p.d by 90° in a capacitor in an RC circuit (capacitors have reactance)



Current **lags** p.d by 90° in an inductor in an LR circuit (inductors have reactance)

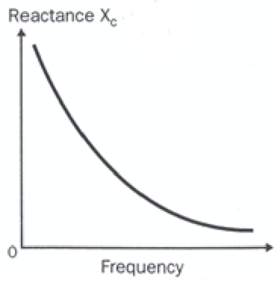


Remember: CIVIL

The reactance of a capacitor is frequency dependant and given by:

$$X_C = \frac{1}{\omega C}$$

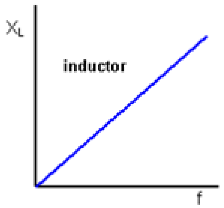
where Xc is the reactance of the capacitor (Ω), C is the capacitance (F) and ω is the angular frequency of the supply voltage (s^{-1}).



Capacitors block LOW frequency.
The reactance of an inductor is also frequency dependant and given by:

$$X_L = \omega L$$

where X_L is the reactance of the inductor (Ω), L is the inductance (H) and ω is the angular frequency of the supply voltage (s^{-1}).



Inductors block HIGH frequency.
Note: The angular frequency ω and the frequency f are related by the equation: $\omega = 2\pi f$

Impedance

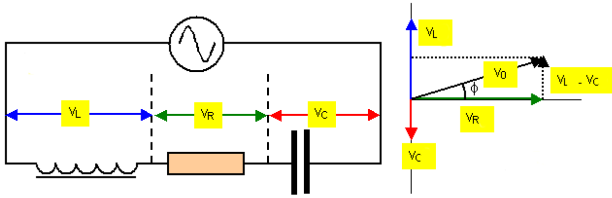
In a circuit containing resistive and reactive components, the current must be calculated by:

$$V = IZ$$

Where Z is the impedance and is calculated by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

(Note: This equation is not given to you at Level 3)



In an RCL circuit, the overall effect on the current is determined by the impedance (a combination of the resistance and the reactance of the capacitor and the inductor).

The alternating current in an RCL series circuit will be at a maximum when $V_L = V_C$. This occurs when X_L and X_C are equal (since they are opposite) and occurs at a unique frequency for each RCL circuit.

This frequency is known as the resonant frequency and can be calculated by the **derivation**:

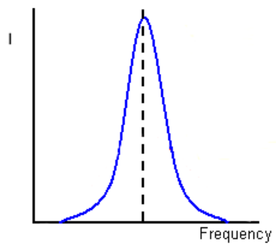
At resonance $X_C = X_L$

$$\frac{1}{\omega C} = \omega L$$

$$\frac{1}{LC} = \omega^2$$

$$2\pi f = \sqrt{\frac{1}{LC}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$



Resonant frequencies are used extensively in telecommunications.