Centre of Mass

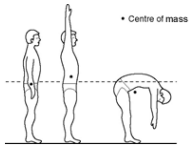
The centre of mass is the point at which all the mass of an object appears to be concentrated. It is the balance point of an object, the place where an object must be supported if it is to remain balanced.

The important conservation law: *The centre of mass remains at rest, or continues to travel at a constant velocity, unless an* ***external force*** *acts on the system.*

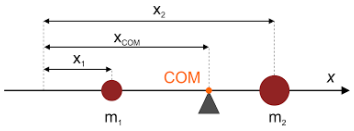
When an external force acts upon a system, the system reacts as though all the matter were concentrated at the centre of mass.

Any external force applied to the body in line with the centre of mass will move the entire system.
Any external force applied to the body which is not in line with the centre of mass will result in a rotation of the system.
The centre of mass can be thought of as a pivot point around which the system can revolve.

The centre of mass of a person is roughly the belly button although its position can be changed slightly by redistributing the mass.

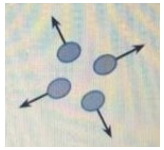


$$x_{COM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



Centre of mass during collisions

The centre of mass of an explosion remains approximately at the point of the explosion.



Linear Momentum

Linear momentum is the product of the mass and velocity of an object. It is a vector quantity, possessing a magnitude and a direction.

$$p = mv$$

Conservation of momentum

If a closed system is not affected by external forces, its total linear momentum does not change. Momentum is conserved in a collision.

Momentum before = momentum after

Change in momentum.

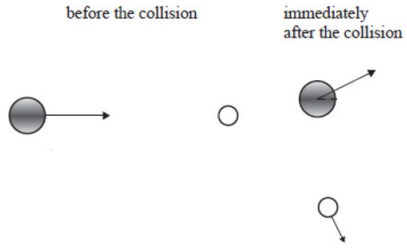
$$\Delta p = F\Delta t$$

The product of force × time is called impulse (AKA change in momentum). Impulse is force multiplied by the time for which it acts. Impulse is a vector. It is not usually given its own symbol. It is measured in Ns.

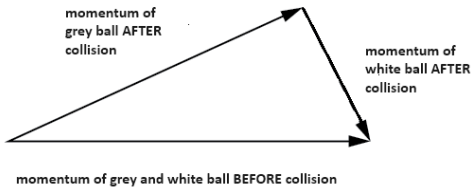
Inertia: The tendency of objects in motion to stay in motion, and objects at rest to stay at rest, unless another force causes its speed or direction to change

Two-dimensional analysis

Momentum is also conserved in a collision where the objects are moving in two dimensions. As momentum is a vector quantity the sum must be considered as a vector sum. Split the momentum up into two components, usually the x and y directions. The total momentum in each of these directions must be conserved. For a collision between two objects:



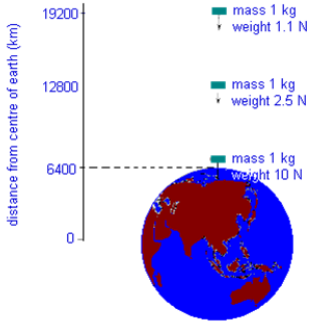
The analysis involves the drawing of a vector diagram:



Trigonometry (SOHCAHTOA and $c^2 = a^2 + b^2$ should be sufficient to solve any physics problems at NCEA Level 3 but – if required – you could always draw a scale diagram or use the sine/cosine rule)

Gravitational Forces

A downward force is produced on all masses near the earth's surface by our planet's gravitational attraction. This downward force on an object is called the object's **weight**.
Mass is the 'quantity of matter' in an object and is a measure of how hard it is to accelerate the object. The **mass** of an object is the same on earth, on the moon or in outer space.
At the earth's surface, the gravitational field strength (g) is about 9.81 N kg^{-1} so an object with a mass of 60 kg, such as a person, has a weight of 60 x 9.81 = 587 N.



Weight is the force exerted on a mass by other masses. The weight of an object depends on what other masses are around. The weight of an object is different on earth, on the moon or in outer space.

Gravity for All

In reality, every mass attracts every other mass.

$$F_g = \frac{GMm}{r^2}$$

We don't notice it for most masses because this force is tiny in size, due to the value of G, the universal constant of gravitation: $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} (*G is a universal constant as it has the same value everywhere in the universe*).

It is only when at least one of the masses involved in the attraction is large that the force becomes noticeable! The earth has a mass of 5.98×10^{24} kg and so exerts a noticeable force even on small masses.

Satellites

Uses of satellites

- Moonlight
- weather forecasting
- studying the atmosphere
- telecommunication
- mapping the Earth

Orbital velocity

The velocity with which an object circles the Earth in a circular path is known as orbital velocity. The centripetal force required to make the satellite go in a circular orbit actually acts towards the centre of the Earth. It is created by the universal gravitational force.

$$F = \frac{GMm}{R^2} \therefore \frac{mv^2}{R} = \frac{GMm}{R^2}$$

$$V^2 = \frac{GM}{R}$$

$$\left[g = \frac{Gm}{R^2}, \therefore Gm = gR^2 \right]$$

Substituting for Gm in the equation, we get $V = \sqrt{\frac{gR^2}{R}} = \sqrt{gR}$

The orbital velocity near the Earth's surface can be calculated taking g = 9.8 ms^{-2} and approximately R = 6.4 x 10^6 m so v = 7.9 x 10^3 $m\,s^{-1}$

Escape velocity

Escape velocity is the velocity with which a body should be projected to enable it to escape from the gravitational influence of the Earth.

$$F = \frac{GMm}{R^2}$$ Integrating:

Gives the gravitational potential as $\frac{GMm}{R}$

$$\frac{1}{2}mV_e^2 = \frac{GMm}{R} \text{ so } V_e^2 = \frac{2GM}{R} \text{ so } V_e = \sqrt{\frac{2GM}{R}}$$

$$\left[g = \frac{GM}{R^2}, \therefore GM = gR^2 \right]$$

$$V_e = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR}$$

If g = 9.8ms^{-2} and R = 6.4 x 10^6m Then v_e = 11.2 x 10^3 ms^{-1}

Circular motion

Every object moving in a circular path must have a net or unbalanced force on it to continually change its direction – this is called centripetal force.

Geostationary Orbit

At an exact value of R, a satellite can be placed to exactly match the rotation of the Earth in what is called a geostationary orbit.

$$F_c = \frac{mv^2}{R} \text{ and } v = \frac{2\pi r}{T}$$

$$F_c = \frac{4\pi^2 mR}{T^2}$$

$$F_c = F_g$$

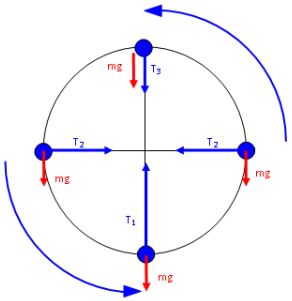
$$\frac{4\pi^2 mR}{T^2} = \frac{GMm}{R^2}$$

rearranging gives:

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

Knowing T = 24 hours, the height of a geostationary satellite is calculated to be 36000km. In order for the satellite to be geostationary, the orbit must also be in the plane of the equator and travelling west to east.

Motion in a vertical circle



If an object is being swung round on a string in a vertical circle at a constant speed the centripetal force must be constant but because its weight (mg) provides part of the centripetal force as it goes round the tension in the string will vary.

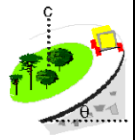
Let the tension in the string be T_1 at the bottom of the circle, T_2 at the sides and T_3 at the top.

At the bottom of the circle: $T_1 - mg = mv^2/r$ so $T_1 = mv^2/r + mg$
At the sides of the circle: $T_2 = mv^2/r$
At the top of the circle: $T_3 + mg = mv^2/r$ so $T_3 = mv^2/r - mg$

So, as the object goes round the circle the tension in the string varies being greatest at the bottom of the circle and least at the top.

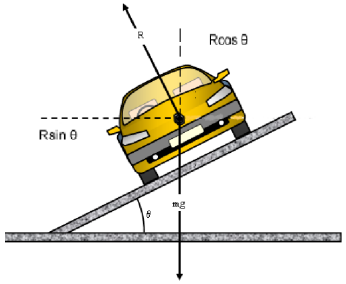
Banked Tracks

A car on a banked curve must experience a centripetal force if it is moving in an arc (part of a circle).



car on banked curve

Looking at it "end-on



The Reaction force can be divided into a vertical and horizontal component. R sin θ provides the centripetal force to make the car move in a horizontal arc/circle. R cos θ must be equivalent to the vertical weight force.

$$R\sin\theta = \frac{mv^2}{r} \quad R\cos\theta = mg$$

$$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2/r}{mg}$$

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

This is the condition for the car to go in a circle. If the car moves faster than this speed, it will slip up the banked curve, and if it goes slower, it will slip down the bank (assuming the road is perfectly frictionless).

This same Physics theory can be applied to conical pendulums and aircraft banking during a turn.

Rotational Motion

Basic Rotational Quantities

Rotation – or Angular motion – can best be described in terms of angular displacement, time, angular velocity, and angular acceleration.

Comparing linear and angular motion

	Linear Motion	Rotational Motion	
Displacement	d	θ	Angular Displacement
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations			Motion equations
	$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$	
	$d = v_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$	
	$v_f^2 = v_i^2 + 2ad$	$\omega_f^2 = \omega_i^2 + 2\alpha\theta$	
Mass (linear inertia)	m	I	Moment of inertia
Newton's second law	$F = ma$	$\tau = I\alpha$	Newton's second law
Momentum	$p = mv$	$L = I\omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	Kinetic energy

Angular Motion

2π radians in a circle or revolution, means that $1rpm = \frac{2\pi}{60} rads^{-1}$

Multiply angular quantity by radius to convert to tangential linear quantity.

$d = r\theta$ $\quad v = r\omega$ $\quad a = r\alpha$

Motion problems can be solved using the 4 equations of motion:

$\omega_f = \omega_i + \alpha t$ $\quad \theta = \frac{\omega_f + \omega_i}{2} t$ $\quad \omega_f^2 = \omega_i^2 + 2\alpha\theta$ $\quad \theta = \omega_i t + \frac{1}{2}\alpha t^2$

You will be given 3 variables and asked to work out a fourth variable.

Choose the equation that only has these four (or the equation without the fifth variable).

Angular velocity is the rate of change of angular displacement and can be described both these relationships:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\omega = 2\pi f$$

Torque or turning force causes angular acceleration.

If we want to make a wheel rotate, we exert a tangential force on its rim. The turning effect of a force or torque or moment of a force:

$$\tau = Fr$$

The angular equivalent of torque is calculated by:

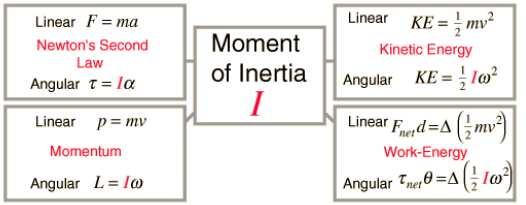
$$\tau = I\alpha$$

Rotational Inertia

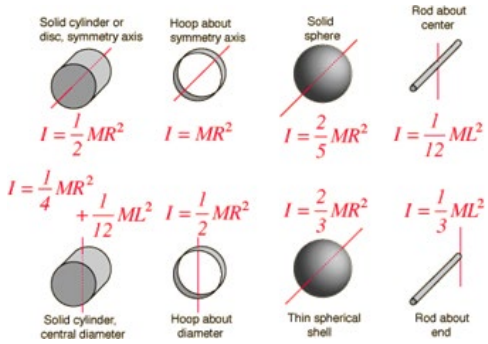
The rotational inertia, I, describes how the mass is arranged around the center of the rotation.

The moment of inertia is $I = \Sigma mr^2$.

The moment of inertia – as the equivalent of mass in linear equations – is fundamental to a number of equations in Angular Mechanics.



The moment of inertia is different for different shapes.

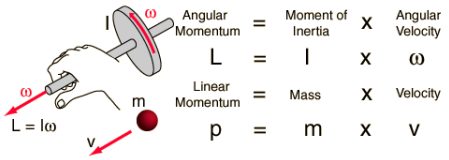


These variations do not need to be remembered – just the idea that they all (pretty much) depend upon r^2.

Angular Momentum and Energy

Angular Momentum

The angular momentum of a rigid object is defined as the product of the moment of inertia and the angular velocity. Similar to linear momentum - angular momentum is conserved if there is no external torque on the object. Angular momentum is a vector quantity. (Unit is kg^2s^{-1})



Angular Momentum	=	Moment of Inertia	x	Angular Velocity
L	=	I	x	ω
Linear Momentum	=	Mass	x	Velocity
p	=	m	x	v

Note – the equation to the right links linear momentum with angular momentum.

$$L = mvr$$

Rotational Kinetic Energy

The rotational kinetic energy of a rotating object is analogous to linear kinetic.

$$KE_{rotational} = \frac{1}{2} I\omega^2$$

$$KE_{rotational} = \frac{1}{2} I\omega^2 \text{ (ω: angular velocity; } I\text{: rotational inertia (moment of inertia))} \quad \text{Linear and rotational kinetic energy have the same form.} \quad \frac{1}{2} mv^2 = KE_{linear} \text{ (v: linear velocity; } m\text{: translational inertia (mass))}$$

Rotational kinetic energy can be stored in flywheel e.g. toy cars

Since E_k depends on I, a hollow cylinder will have greater Rotational E_K than a solid cylinder.

$$PE_{gravity} = KE_{translational} + KE_{rotational}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

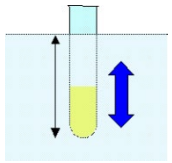
This means that a solid cylinder of same mass will roll down a slope faster than a hollow cylinder since more gravitational E_P is converted to Rotational E_K (for the hollow cylinder).

Simple Harmonic Motion

Any motion that repeats itself after a certain period is known as a **periodic** motion, and since such a motion can be represented in terms of sines and cosines it is called a **harmonic** motion.

Simple harmonic motion (S.H.M. for short) is the name given to a particular type of harmonic vibration. The following are examples of SHM:

- a test-tube bobbing up and down in water
- a simple pendulum
- a compound pendulum
- a vibrating spring
- a vibrating cantilever
- a marble on a concave surface
- liquid oscillating in a U-tube



SHM requires the same magnitude of force irrespective of direction of displacement but is a restoring force dependent on the amount of displacement, y, from the equilibrium position:

$$F = -ky$$

The equation for the acceleration of a body undergoing SHM is usually written as

$$a = -\omega^2 y$$

The solution to this equation:

$$y = A\sin\omega t$$

Since the equation for acceleration can be obtained by twice differentiating the equation for displacement against time, we get the top set of equations (measured with respect to the equilibrium position).

$$y = A\sin\omega t \qquad v = A\omega\cos\omega t \qquad a = -A\omega^2\sin\omega t$$

$$y = A\cos\omega t \qquad v = -A\omega\sin\omega t \qquad a = -A\omega^2\cos\omega t$$

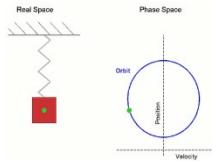
There are alternative equations for SHM based on measurements from the extremities of the swing (as shown by the bottom set of equations).

To solve for a particular time, t, you need radians (but the maximum values are

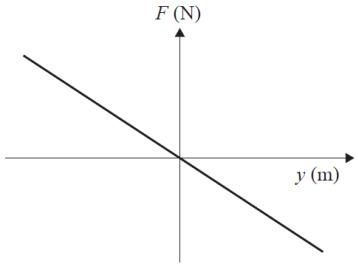
$$y = A \qquad v = A\omega \qquad a = -A\omega^2$$

Where: $T = \frac{2\pi}{\omega}$

You can also solve using a reference circle.



Graphing SHM



The gradient = - k

The simple pendulum

The restoring force F is the component of the weight of the bob. Therefore

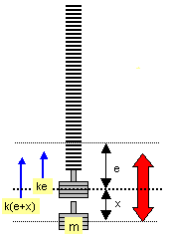
F = - mg sin θ = ma giving a = - g sin θ
But for small angles sin θ tends to θ^c, and therefore a = -g θ = - gx/L
The value of ω^2 is g/L, and so the period of a simple pendulum is:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

This formula is only accurate for small angles of swing, however.

The spring-mass system

The mass will oscillate due to both the effect of the gravitational attraction (mg) and the varying force in the spring (k(e + x)). At any point distance x from the midpoint:
restoring force = k(e + x) - mg



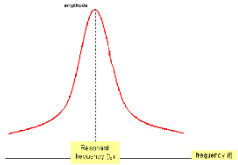
But F = ma, so ma = - kx and this shows that the acceleration is directly proportional to the displacement.

From the defining equation for SHM ($a = -\omega^2 x$) we have $\omega = k/m$ and therefore the period T is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Resonance

All systems have their own natural frequency, and if you apply a driving force of the same frequency and in phase with the initial oscillations then resonance results, the amplitude of the oscillations gets larger and larger.



Parts of a car may vibrate if you drive over a bumpy road at a speed where the vibrations transmitted to the body are at the resonant frequency of that apart. Bass frequencies from stereo speakers can make a room resonate.

Damped oscillations

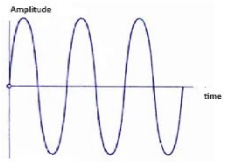
The amount of damping of a system affects the shape of the resonance curve.

Damped oscillations are oscillations where energy is taken from the system and so the amplitude decays. They may be of two types:

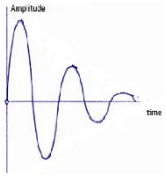
(i) Natural damping, examples of which are: internal forces in a spring, fluids exerting a viscous drag.

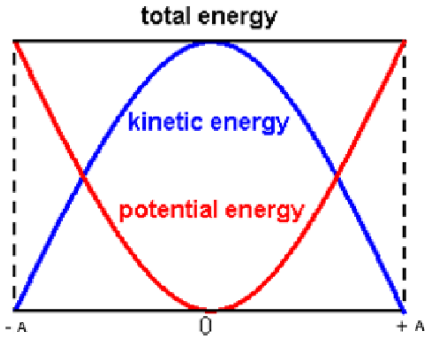
(ii) Artificial damping, examples of which are: shock absorbers in cars, interference damping - gun mountings on ships.

An undamped system:



A damped system:



Energy in simple harmonic motion

A graph of the variation of potential energy, kinetic energy and the total energy