All you wanted to know about Formulae and Graphs (but were afraid to ask!) - The Advanced version

To be able to carry out practical investigations in Physics you must understand the following:

1. What variables you investigating
2. How the formula relates to the variables you are investigating
3. How the formula relates to a "y = mx + c" graph
4. How to process the variables and draw a linear graph (including uncertainties)
5. What the linear graph actually tells you and verifying the relationship.

Both the following are simple examples.

Example 1:

Aim: To find the mathematical relationship between the **distance travelled, d**, and the **time taken, t**, for a basketball to fall from rest towards the ground.

Step 1: What variables are you investigating?

Distance travelled, d, and the **time taken, t** (read the aim)

Step 2: How the formula relates to the variables you are investigating

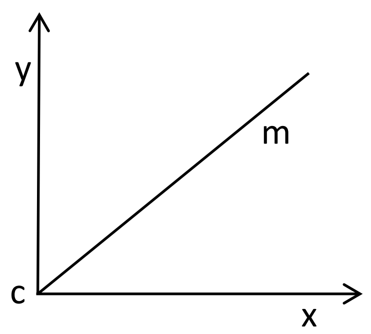
Formula: $d = v_i t + \frac{1}{2} g t^2$

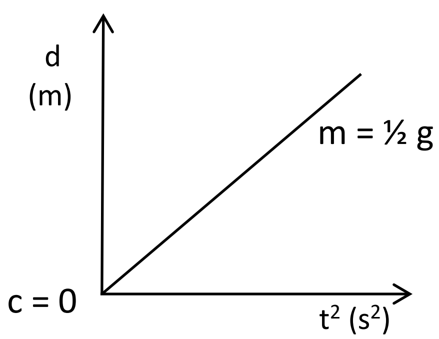
Where v_i = initial velocity (zero) **d** = distance, **g** = acceleration due to gravity and **t** = time

Step 3: How the formula relates to a "y = mx + c" graph

Since $y = mx + c$ and $d = \frac{1}{2} g t^2$ (forget the $v_i t$ - *it equals zero)*

The $y = mx + c$ *graph should a straight line graph and so should* $d = \frac{1}{2} g t^2$ ..... providing you plot ***d*** on the y-axis and t^2 on the x-axis)





Providing you plot ***d*** on the y-axis and t^2 on the x-axis – this should give you an intercept of 0 and the gradient, m, should be equal to $\frac{1}{2} g$, the acceleration due to gravity, g.

Step 4: How to process the variables and draw a linear graph (including uncertainties)

Now you understand that the graph is ***d*** versus t^2, calculate values for t^2 (units of s^2) and plot the graph. You also need to calculate appropriate uncertainties for t^2 by 2 x % uncertainty of t.

Step 5: What the linear graph actually tells you and verifying the relationship

You should be able to write a mathematical formula for your graph e.g. ***d = 4.7(±0.2) t²*** (again based upon y = mx + c where c = 0).

In this case, using your mathematical equation ***d = 4.7(±0.2) t²***

and the theoretical equation ***d = ½ g t²*** to verify acceleration due to gravity (g= 9.81 ms^{-2}).

$$d = 4.7 \pm 0.2 \quad t^2$$
$$d = ½ \quad g \quad t^2$$

The above shows us that 4.7 ± 0.2 = ½ g

g = 2 x (4.7 ± 0.2)

g = 2 $\pm 0\%$ x (4.7$\pm$4%)	[2 has an uncertainty of 0%]
$g = 9.4\ \pm 4\%$	[0% + 4% = 4% uncertainty]
$g = 9.4\ \pm 0.4$	[Round uncertainty to 1SF]
$g = 9.4\ \pm 0.4ms^{-2}$	[Final answer, include units]

Does the experiment verify g? 9.4 $\pm$ 0.4 is 9.0 to 9.8 so not quite but almost….

Example 2:

Aim: To find the relationship between the **frequency** of the note sounded in a closed column of air and the **length** of the closed air column.

Step 1: What variables are you investigating?

Frequency of the note sounded in a closed column of air and the **length** of the closed air column

Step 2: How the formula relates to the variables you are investigating

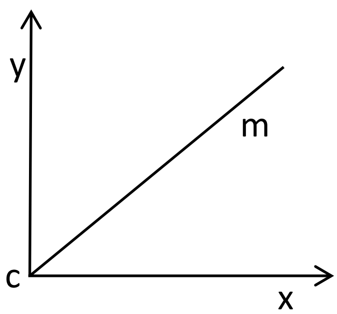
Formula: f = 4v/L

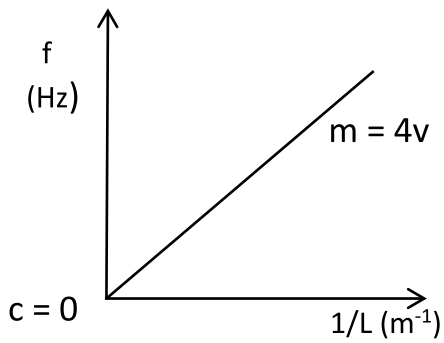
Where **f** = frequency, **v** = velocity of sound in air and **L** = length of the closed air column

Step 3: How the formula relates to a "y = mx + c" graph

Since **y = mx + c** and **f = 4v/L**

The **y = mx + c** *graph should a straight line graph and so should* **f = 4v/L** ….. providing you plot **f** on the y-axis and **1/L** on the x-axis)





Providing you plot **f** on the y-axis and ***1/L*** on the x-axis – this should give you an intercept of 0 and the gradient, m, should be equal to **4 v**, 4 x the velocity of sound in air.

Step 4: How to process the variables and draw a linear graph (including uncertainties)

Now you understand that the graph is **f** versus ***1/L***, calculate values for ***1/L*** (units of m^{-1}) and plot the graph. You also need to calculate appropriate uncertainties for 1/L by 1 x % uncertainty of L.

Step 5: What the linear graph actually tells you and verifying the relationship

You should be able to write a mathematical formula for your graph e.g. **f = 1300(±100)/L** (again based upon y = mx + c where c = 0).

In this case, using your mathematical equation **f = 1300(±100)/L** and the theoretical equation **f = 4v/L** to verify velocity of sound in air (v= 340 ms^{-1}).

$$f = 1300 \pm 100 / L$$

$$f = 4v / L$$

The above shows us that 1300 ± 100 = 4v

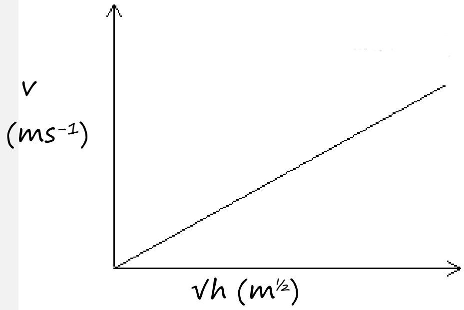
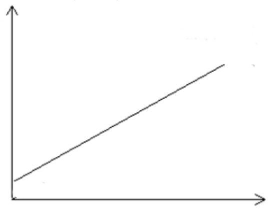
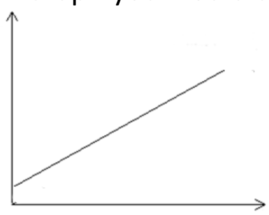
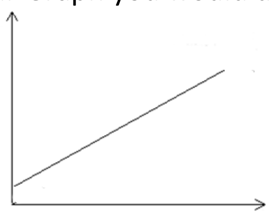
v = 1300 (± 100) x 1/4

v = 1300 (± 8%) x $(1/4 \pm 0\%)$	[1/4 has an uncertainty of 0%]
$v = 325 \pm 8\%$	[0% + 8% = 8% uncertainty]
$v = 325 \pm 26$	[Round uncertainty to 1SF]
$v = 325 \pm 26 ms^{-1}$	[Final answer, include units]

Does the experiment verify v?

325 ± 26 is 299 - 351 and since the speed of sound in air is 340 ms^{-1} – YES.

Complete the following:

1.	Aim: To find the mathematical relationship between the **velocity, v,** of a ball when it hits the ground, and the **height, h**, it is dropped from. Formula: $v = \sqrt{2gh}$ What do you have to process? **height, h** to **√height, √h** √h has unc. of **½ x % unc.** of h	Variables: **velocity, v** **height, h**	Linear Graph you would draw:  What does the gradient tell you? **(gradient) m = √2g**
2.	Aim: To find the mathematical relationship between the rotational kinetic energy, E_{krot}, and the angular velocity,ω, of an object Formula: $E_{krot} = ½I\ \omega^2$ What do you have to process?	Variables:	Linear Graph you would draw:  What does the gradient tell you?
3.	Aim: To find the mathematical relationship between the **Time period, T, of a pendulum** and its **length, l**. Formula: $T = 2\pi\sqrt{\frac{l}{g}}$ What do you have to process?	Variables:	Linear Graph you would draw:  What does the gradient tell you?
4.	Aim: To find the mathematical relationship between the **time period, T,** of an uneven two spring system and the **mass, m,** suspended from the springs. Formula: $T = \pi\sqrt{\frac{m}{k_1}} + \pi\sqrt{\frac{m}{k_2}}$ (In this case, the spring with a much lower value of k, e.g. k_2 will only affect the intercept and not the gradient) What do you have to process?	Variables:	Linear Graph you would draw:  What does the gradient tell you?

Answers

	formula	Variables	y-axis	x-axis	gradient
1.	$v = \sqrt{2gh}$	h v	v (ms^{-1})	$\sqrt{h}$ ($m^{½}$)	$\sqrt{2g}$
2.	$E_{krot} = ½I\,\omega^2$	E_{krot} ω	E_{krot} (J)	ω^2 (s^{-2})	½I
3.	$T = 2\pi\sqrt{\frac{l}{g}}$	T l	T (s)	$\sqrt{l}$ ($m^{½}$)	$2\pi / \sqrt{g}$
4.	$T = \pi\sqrt{\frac{m}{k_1}} + \pi\sqrt{\frac{m}{k_2}}$	T m	T (s)	$\sqrt{m}$ ($kg^{½}$)	$\pi / \sqrt{k_1}$