# ELECTRICITY: AC QUESTIONS

QUESTION ONE (2019;1)

Mobile contactless payment systems are used in shops and restaurants throughout New Zealand. The circuit inside the contactless card does not have its own power source. It is powered by induction using a pair of coils: one in the contactless card and the other in the payment machine. A simplified model is shown in the diagram.

(c)   The payment machine's induction coil has an alternating current of frequency 13.6×10^6 Hz. Using physics principles, explain how a voltage is induced in the coil of the contactless card when it is placed near the coil of the payment machine (no calculations required).

(d)   At the frequency 13.6×10^6 Hz, the contactless card's induction coil has a reactance of 427 Ω. The contactless card circuit contains a capacitor in series with the coil that causes the circuit to resonate only at this frequency.
• State the conditions under which resonance occurs.
• Calculate the capacitance that is needed for resonance.
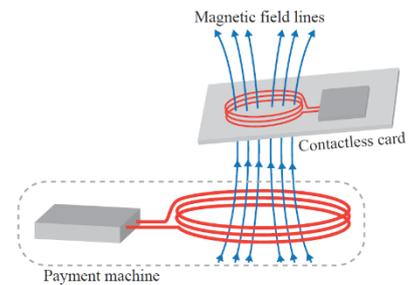
QUESTION THREE (2018;3)

Casey is experimenting with building inductors and capacitors. Casey makes a capacitor of capacitance 9.45×10^{-7} F.

To make an inductor, Casey winds several hundred turns of insulated copper wire around an iron rod. Casey wants to test its inductance. Casey connects the circuit shown below. The voltage across the lamp is measured to be 4.64 V_{RMS}, and across the inductor to be 11.1 V_{RMS}.
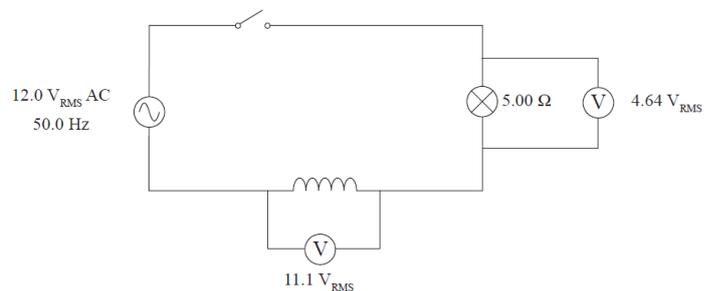
(b)   Show that the inductance of Casey's inductor is 3.81×10^{-2} H.

Casey adds in the 9.45×10^{-7} F capacitor to create an LCR series circuit. The light bulb barely glows. Casey switches the AC power supply to its maximum frequency setting of 4.00×10^2 Hz.

(c)   Determine the new impedance of the circuit.

(d)   Calculate the resonant frequency for the circuit and compare this with the maximum frequency setting of the power supply and describe how Casey could physically alter the inductor and capacitor to increase the current through the light bulb using this power supply.

QUESTION THREE (2017;3)

It is important that the wood used in buildings does not have much water in it. Thomas uses a parallel-plate capacitor, with the wood as the dielectric, to measure the water content of the wood. Water has a higher dielectric constant than wood. One way of measuring the water content in the wood is by using the circuit shown. Thomas connects the circuit, and makes the following measurements:

|	
Supply voltage	$= 12.0 \text{ V}_{rms}$
Frequency	$= 151 \text{ Hz}$
Resistance of the resistor	$= 50.0 \ \Omega$
Reactance of capacitor	$= 23.5 \ \Omega$
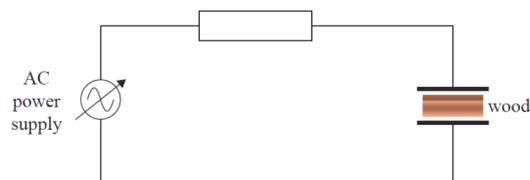
(a)   Calculate the peak voltage of the AC power supply.

(b)   Calculate the rms current in the circuit.

(c)   Explain what would happen to the circuit current when the wood in the capacitor is replaced by a similar piece of wood that contains more water.

(d)   An inductor is added in series with the capacitor and resistor in the circuit. The reactance of the inductor is 35.7 Ω at 151 Hz. The reactance of the capacitor is 23.5 Ω at 151 Hz. Thomas adjusts the frequency until the current is maximum. Calculate the resonant frequency and explain why the current is maximum at the resonant frequency.

MEASURING IRON IN SAND (2016;3)

Vivienne wants to measure the amount of iron in iron-sand mixtures collected from different beaches. The diagram below shows the circuit that she uses. The circuit includes a 500-turn coil with a resistance of 15.0 Ω, and an AC supply. The coil behaves like a resistor and an inductor in series. The coil has a hollow core that is initially empty. Vivienne adjusts the power supply voltage to 6.0 V rms.

6.00 V rms

A

15.0 Ω
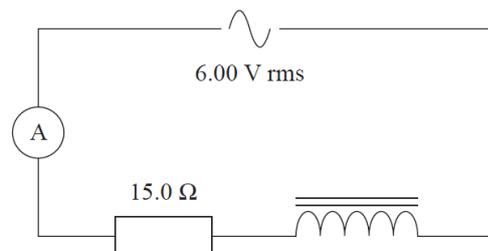
(a)   Calculate the instantaneous maximum (peak) voltage across the power supply.

(b)   During testing, Vivienne puts a mixture of iron and sand inside the core of the coil. State what effect this has on the size of the coil's reactance. With reference to impedance, explain what happens to the size of the current in the circuit as she adds the mixture of iron and sand.

(c)   When Vivienne sets the frequency of the current to 1.00×10^3 Hz, the inductance of the coil is 3.18×10^{-3} H. Using a phasor diagram or otherwise, calculate the size of the rms current in the circuit.
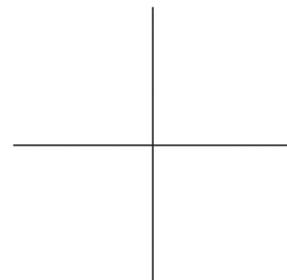
(d)   Vivenne adds a capacitor in series with the coil and finds that the current increases. Explain why the current increases.

AC CIRCUITS (2015;1)

An AC circuit has a variable capacitor, an inductor, and a resistor in series, as shown.
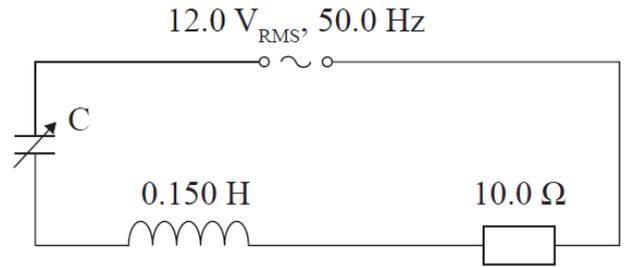
12.0 V$_{RMS}$, 50.0 Hz

C

0.150 H   10.0 Ω

(a) Calculate the angular frequency of the supply.

(b) Show that the reactance of the inductor is 47.1 Ω.

(c) When the variable capacitor has a value of 100 x 10^{-6} F, the voltage across the capacitor is measured as 20.9 V$_{RMS}$ and the current flowing in the circuit is measured as 0.656 A$_{RMS}$. Calculate the voltages across the inductor and the resistor, and draw labelled phasors showing the voltages across the capacitor, the inductor, and the resistor.

(d) The variable capacitor is adjusted so that the circuit is now at resonance. Explain, using physical principles, why the current is now a maximum, and calculate the value of the current in the circuit at resonance.

AC (2014;1)
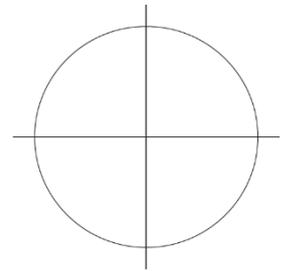
3000 turns   600 turns

The ideal transformer shown has 3000 turns in its primary coil, and 600 turns in the secondary coil. A 240 V$_{rms}$ AC power supply is connected across the primary coil. The secondary coil is connected to an external circuit.

240 V$_{rms}$ external circuit
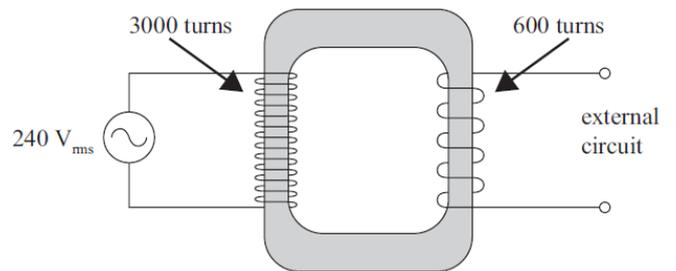
(a) (i) Calculate the rms voltage across the external circuit.

(ii) Calculate the peak voltage across the external circuit.

R   3.69 Ω   L   16.5 mH

(b) Explain why rms values are often used to describe AC voltages.

(c) The external circuit consists of a resistor and an inductor as shown. The frequency of the power supply is 50.0 Hz. By drawing a phasor diagram, show how the impedance of the external circuit can be calculated.

R   3.69 Ω   L   16.5 mH

(d) A capacitor is added to the external circuit, causing the circuit to be at resonance. Determine the rms voltage across the capacitor.

C
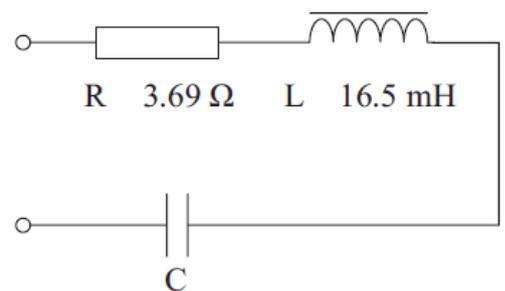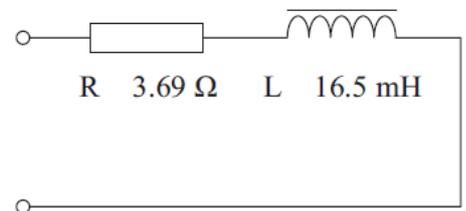
Using a capacitor and an inductor to produce a burst of sound (2013;3)

In the circuit below, the speaker will produce a sound that will depend on the magnitude and frequency of the current through it. The frequency of the supply is set to 4.50×10^2 Hz. At this frequency, the total impedance of the circuit is 93.0 Ω.

25.0 V$_{rms}$ variable frequency

15.0×10^{-6} F

L

55.0 Ω

speaker
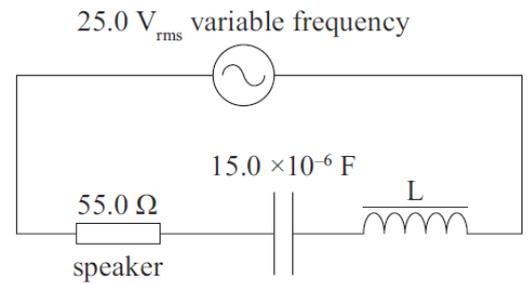
(a)   Calculate the reactance of the capacitor at this frequency.

(b)   The supply voltage leads the current by an angle θ. Calculate the value of θ.

(c)   Show that the reactance of the inductor at this frequency is 98.6 Ω.

(d)   Explain what must be done to the frequency of the supply to bring the circuit to resonance.

(e)   The resonant frequency is 2.20×10^2 Hz. By considering the resonance condition or any other method, calculate the inductance of the inductor.

(f)   Explain how the frequency of the supply can be altered to produce a short burst of sound from the speaker.


THE RECEIVER (2012;3)

The car's remote control sends out radio waves of frequency 27.0 MHz (27.0×10^6 Hz). Hugo starts investigating the car's radio receiver by removing an inductor coil of inductance 1.00×10^{-6} H.

47.0 Ω

27.0 MHz
5.00 V$_{rms}$

1.00×10^{-6} H

(a)   Show that the reactance of the inductor at this frequency is 170 Ω.

Hugo connects the inductor in series with a 47.0 Ω resistor.

(b)   Calculate the rms current through the resistor when this arrangement is connected to a

5.00 V$_{rms}$ AC supply oscillating at 27.0 MHz.

(c)   A capacitor can be added in series to cause this circuit to resonate. By stating the conditions under which resonance occurs, calculate the capacitance needed to bring the circuit to resonance at this frequency.

(d)   When two radio-controlled cars are sold as a set, the second controller uses a 49.0 MHz wave to avoid interference between the two radio signals.
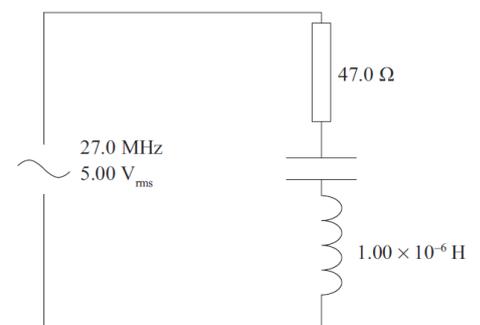
Explain why a circuit built to resonate at 27.0 MHz does not respond to a 49.0 MHz signal. As part of your answer, determine the current in the circuit in part (c) when the supply frequency is 27.0 MHz and when the supply frequency is 49.0 MHz.

PHASE DIFFERENCE IN AC CIRCUITS (2011;2)

A teacher demonstrates resonance of AC circuits with a resistor connected in series to an inductor, a capacitor and an AC supply, as shown in the diagram. She uses a computer to track current and voltage, making the following measurements:

|	Peak value
Supply voltage, V_S	10 V
Voltage across the capacitor, V_C	1.0 V
Voltage across the inductor, V_L	10 V
Current, I	0.30 A
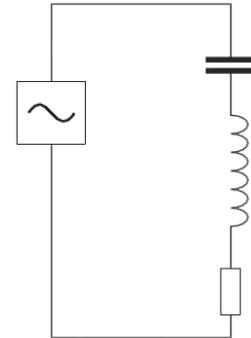
(a)   Calculate the impedance of the circuit.

The computer shows the following graphs:

(b)   Use data from the table and the graphs to calculate the capacitive reactance and hence the capacitance of the capacitor.

(c)   Draw a phasor diagram, approximately to scale, to show the phase relationship between the voltage across the capacitor, the voltage across the inductor, and the supply voltage. Using your diagram, or any other method, determine the phase difference between the supply voltage and the current.

(d)   Describe how the frequency should be changed to make the circuit come to resonance and explain your reasoning.
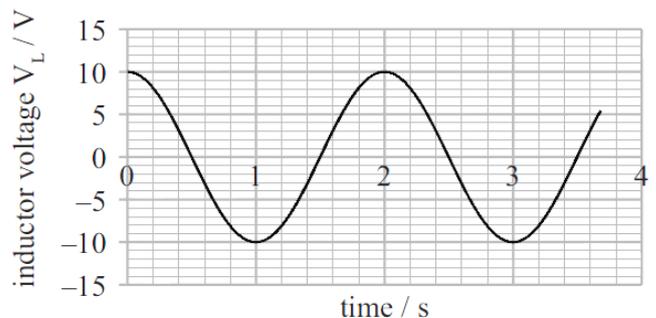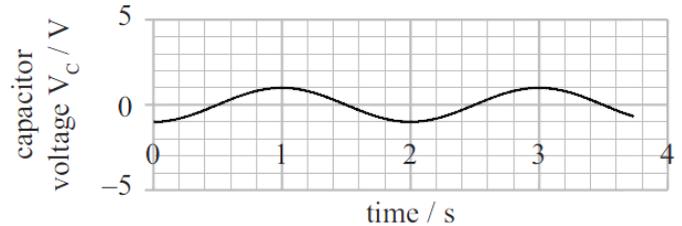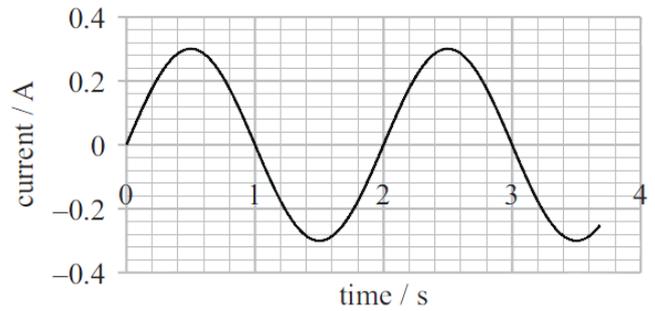
CAPACITANCE MOISTURE METER (2010;2)

The moisture content of wood can be tested by measuring capacitance. A simple moisture meter is based on two parallel metal plates. The test sample fits between the plates. A museum wants to develop a circuit that will be much more sensitive to change in moisture content. To this end, the capacitor is included as part of an LCR circuit.

wood sample
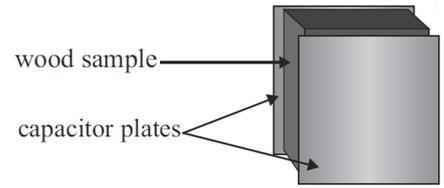
capacitor plates

Sensitive detector unit

variable frequency ac supply, $V_{rms} = 16.0$ V
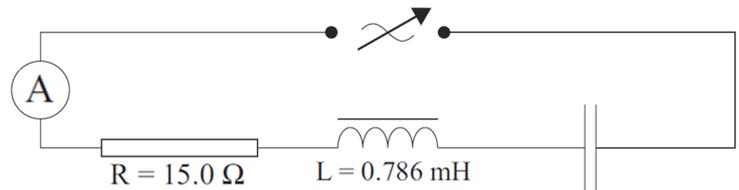
(c) Show that the resonant frequency of the circuit depends on L and C according to the equation:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

(A)

$R = 15.0\ \Omega$ $L = 0.786$ mH

(d) A wood sample is placed between the capacitor plates. Describe how you could find the resonant frequency of the circuit as you varied the frequency of the AC supply.

(e) Wood with a moisture content of 15% has an ε_r of 16.8.

Show that an AC supply set at 1.00×10^6 Hz will bring the circuit to resonance for a sample of wood with this moisture content.

(f) The moisture content of the wood sample rises such that its relative permittivity increases to 16.9. The AC supply remains unchanged at 1.00×10^6 Hz. Show that you would expect the ammeter to read approximately 0.5 A.

(g) In the above example, the relative permittivity of the wood increases from 16.8 to 16.9. Compare the observed change this causes on the simple moisture meter with that on the sensitive detector, to show that the second circuit is more sensitive.

MODELLING SPEAKERS (2009;1)

In a stereo sound system, the sound is produced by speakers. Some stereo systems have three types of speaker. To model these speakers, Derek builds several circuits to study how the current varies with frequency. He uses an 8.0 Ω resistor instead of a speaker. In all his experiments the voltage of the power supply remains constant.
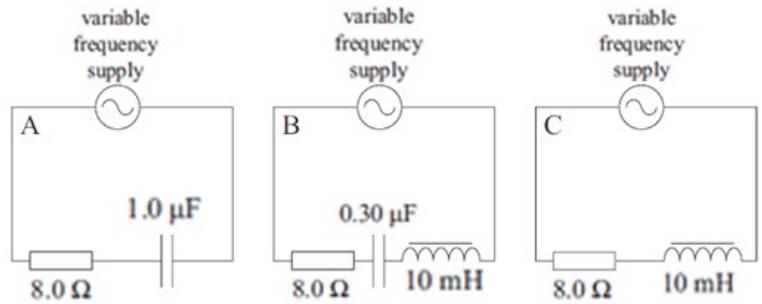
(a) Calculate the reactance of the capacitor in circuit A when the frequency is 2.00 x 10^2 Hz.

(b) Describe and explain how increasing the frequency of the supply will affect the current in circuit A.

(c) Which circuit will produce the largest current when connected to a very low frequency (0.10 Hz) signal? Explain your reasoning.

(d) Calculate the frequency at which the current generated in circuit A is the same as the current in circuit C (The voltage of the supply is the same in both cases). Explain your reasoning clearly.

(e) In circuit B there is one frequency, 2.9 kHz, at which the supply voltage is in phase with the current.

    (i) Draw a phasor diagram to show the relationship between the voltages across the resistor, capacitor and inductor.
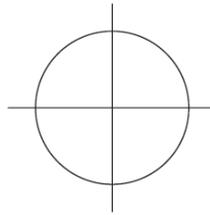

    (ii) Explain how the phase difference between the current and the supply voltage will change as the supply frequency is increased from 100 Hz to 10 kHz. Use phasor diagrams to illustrate your answer.
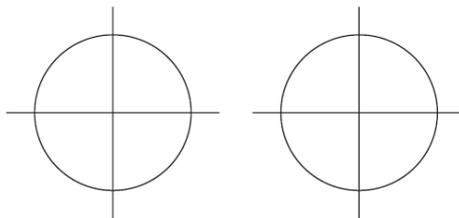
THE INDUCTION COOKER (2008;2)

Sam has an induction cooker and wants to find out how it works. After doing some research, Sam finds that it operates by having a coil of wire underneath an insulating surface. A high frequency alternating current is passed through the coil with a frequency of 27.0×10^3 Hz.
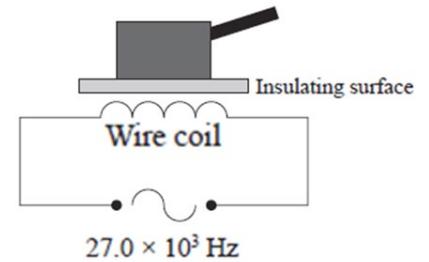
(a)   The arrangement can be used to generate heat within a metal pan placed above the coil. Explain how this occurs.

The coil used in a particular induction cooker is found to have an inductance of 1.30 mH.

(b)   Show that when the frequency of the alternating current is 27.0×10^3 Hz, the reactance of the coil is 221 Ω.

In reality the inductor is part of an LCR circuit as shown below. The resistor has a resistance of 70.0 Ω and the capacitor has a reactance of 358 Ω at 27.0×10^3 Hz.
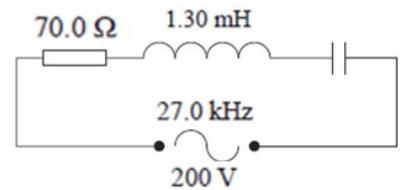
(c)   Show that the capacitance of the capacitor is 1.65×10^8 F.

The phasor diagram shows the phasor representing the resistor voltage.

(d)   On the phasor diagram sketch the phasors representing:
        (i)   the capacitor voltage
        (ii)   the inductor voltage
   (iii)   the supply voltage
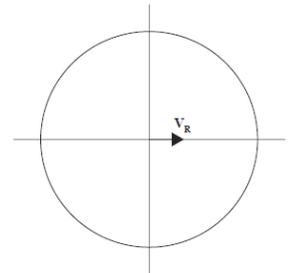
        (Distances and angles should show the approximate relative size and direction of each phasor.)

(e)   Calculate the r.m.s current flowing in this LCR circuit if the r.m.s value of the supply voltage is 200 V.
(f)   Placing an iron saucepan on the hob increases the heating effect of the coil. Explain how an iron cooking pan close to the coil alters the inductance of the coil.
(g)   The iron saucepan has the effect of bringing the circuit to its resonance condition. Describe the condition for resonance in an AC circuit.
(h)   Calculate the r.m.s current in the circuit at resonance.

INDUCTORS AND AC CIRCUITS (2007;2)

Traffic lights can be controlled by using an inductive loop to detect the presence of a car on the road. The loop is a large coil of wire embedded under the road surface. When a car stops over the loop, the inductance of the loop changes. This is sensed by an electrical circuit that causes the traffic lights to change from red to green.
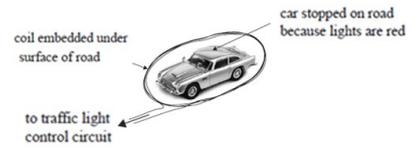
The inductance of the coil of wire must be measured. A possible way of doing this is to use a circuit like the one below. The inductor, L, in the circuit models the coil of wire under the road. The inductor is now connected into the circuit below to model the traffic light control circuit.
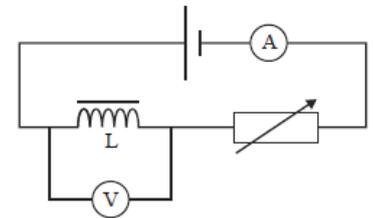
(c)   In the space below draw and label phasors to show the voltages across the inductor, the capacitor and the resistor.

15 V_{RMS}, 81.6 Hz supply

L 18 Ω 2.00×10^{-4} F
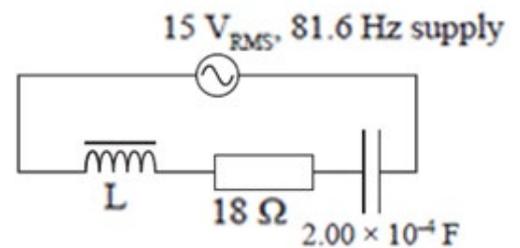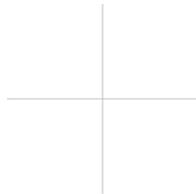
(d)   If the reactance, X_L, of the inductor is smaller than the reactance, X_C, of the capacitor, would the supply voltage phasor lead or lag the current phasor? Explain your answer.

(e)   The frequency of the supply i s 81.6 Hz. Calculate the angular frequency of the supply.

(f)   Show that the reactance of the capacitor is 9.75 Ω.

(g)   If the reactance of the inductor is 1.65 Ω, calculate the current in the circuit.

When a car stops on the road above the coil, the inductance of the coil increases causing the circuit to approach resonance.

(h)   Why does the inductance increase when a car is standing above the coil?

(i)   Explain how this increase in inductance will change the current in the circuit.

(j)   Calculate the value of the current in the circuit at resonance.

(k)   Calculate the inductance of the inductor that would bring the circuit to resonance.

Assuming the energy lost from the resistor is small enough to be ignored, the energy stored in the circuit oscillates between being totally stored in the capacitor and being totally stored in the inductor.

(l)   Calculate the maximum energy stored in the capacitor at resonance.

energy

(m)   On the axes, sketch graphs to show how the energy stored in the capacitor and the energy stored in the inductor change for one complete period of the alternating voltage. Label each graph line.

time (ms)
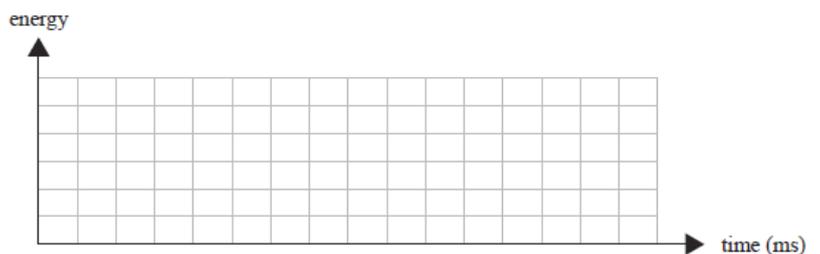
Assume the capacitor is fully charged at t = 0. Show one non-zero value on the time axis. Ignore the energy loss from the resistor.

REED SWITCH (2006;3)

A reed switch is operated by a changing magnetic field. One way to change the magnetic field is to change the current in an inductor. This could be done using the circuit below.

In the circuit below, the 12 V supply has a frequency of 5.0×10^1 Hz.

$12\ V_{rms}$, 50 Hz

$8.5\ \Omega$

8.3×10^{-2} H
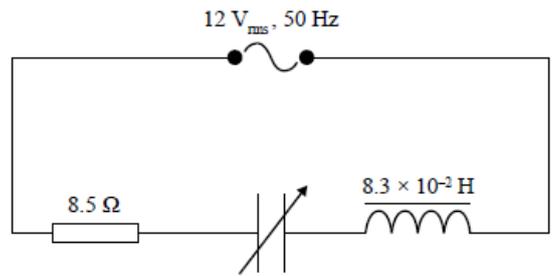
(a)   Calculate the angular frequency of the supply.
(b)   Show that the reactance of the inductor is 26 Ω
(c)   The current in the circuit is 0.42 A. Calculate the reactance of the capacitor.
(d)   Explain how changing the capacitance of the capacitor affects the current in the circuit.
(e)   Calculate the current in the circuit at resonance.

WIND POWER (2005;2)

Jill is making a model wind turbine. It includes a generator constructed from a strong horseshoe magnet and a coil of wire, with 500 turns. The poles of the horseshoe magnet produce a uniform magnetic field with a magnetic flux density of 0.21 T. The coil has an area of 5.20×10^{-3} m², and, on a windy day, completes 2.0 rotations per second.

B
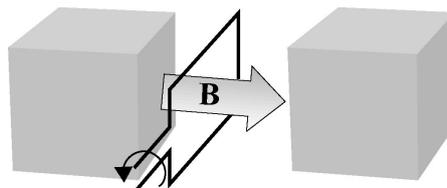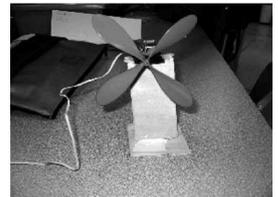
(d)   Calculate the maximum magnetic flux through the coil.

(e)   Calculate the average voltage induced in the coil when the coil rotates from a position of maximum flux to the first position of zero flux.

The graph shows how the voltage induced in the coil changes with time when the coil completes 2.0 rotations per second. On a particularly windy day, one gust of wind doubles the rate of rotation of the coil.

(f)   On the same axes, sketch the shape of the voltage against time graph when the rate of rotation of the coil has doubled.
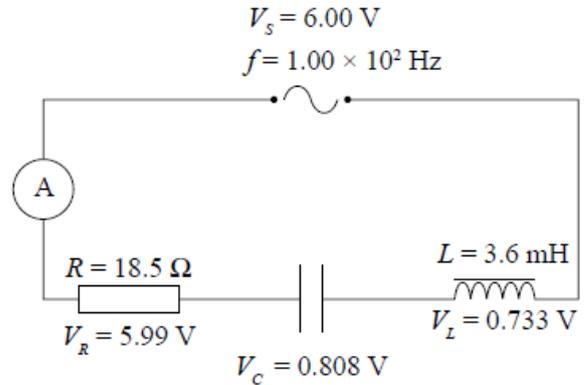
Voltage (V)

Time (s)
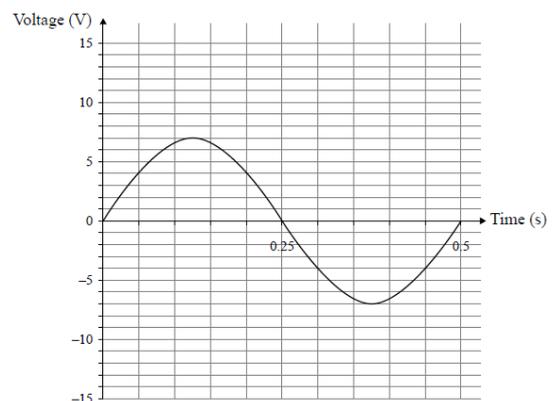
SECURITY SYSTEM (2005;3)

As part of the security measures at a political meeting, a portable metal detector has been installed. The detector includes an AC supply, producing an r.m.s voltage of 6.00 V at a frequency of 100 Hz.

(a)   Calculate the peak voltage of the AC supply.

The AC supply is connected in series with an ammeter, resistor, capacitor and inductor, as shown in the diagram (all voltages are r.m.s values).

$V_s = 6.00$ V
$f = 1.00 \times 10^2$ Hz

(b)   Explain why the numerical values of V_R, V_C and V_L in the diagram do not add up to the numerical value of the supply voltage.

(c)   Show that the ammeter reading in the circuit is 324 mA.

$L = 3.6$ mH

$R = 18.5 \, \Omega$

$V_R = 5.99$ V

$V_L = 0.733$ V

$V_C = 0.808$ V

(d)   Show that the capacitance of the capacitor is 638 x 10⁻⁶ F.

(e)   State the mathematical condition for resonance and use it to show that the resonant frequency of this circuit is 105 Hz.

The main frame of the detector is the inductor in the circuit. When someone walks through the detector they are, temporarily, the core of the inductor.

(f)   Knowing that the resonant frequency is slightly higher than the supply frequency, explain how the reading on the ammeter would show that there was metal in the person's pocket.

AC CIRCUITS (2004;2)

Ana and Craig were investigating AC circuits. They constructed an AC circuit with a lamp and capacitor in series as shown.
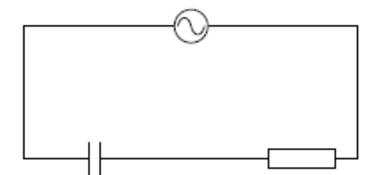
Supply voltage = 12.0 V rms, 50.0 Hz
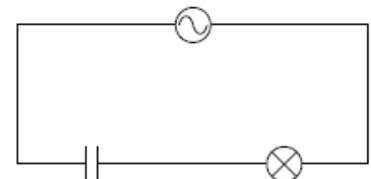
(a)   Explain why the capacitor in this AC circuit allows the lamp to glow continuously but would not do so if connected into a DC circuit.

$C = 2.20 \times 10^{-4}$ F

The students replaced the lamp with a resistor of resistance 16.0 Ω as shown.

Supply voltage = 12.0 V rms, 50.0 Hz

(b)   The diagram below shows the phasors for:
   •   the voltage, V_C , across the capacitor,
   •   the voltage, V_R , across the resistor,
   •   the voltage, V_S , across the supply.

Label the phasors correctly.

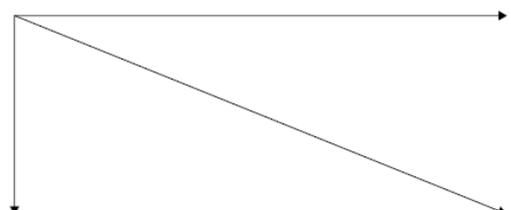$C = 2.20 \times 10^{-4}$ F $R = 16.0 \, \Omega$

Diagram is not to scale.

(c)   Show that the angular frequency of the supply voltage of this circuit is 314 rads^{-1}.
(d)   Calculate the impedance of this circuit.

In a further investigation of AC circuits, Ana and Craig added an ideal inductor to the series circuit.

Supply voltage = 12.0 V rms, 50.0 Hz

(e)   Calculate the inductance of the inductor that will make this a resonant circuit.
(f)   Explain fully what is meant by a resonant circuit.
(g)   Calculate the value of the r.m.s. current in the circuit when it is at resonance.

$C = 2.20 \times 10^{-4}$ F $R = 16.0\ \Omega$

(h)   On the axes below, sketch a graph of rms current against frequency for the circuit. Indicate appropriate current and frequency values.

I_{rms} (A)

frequency (Hz)