Level 3 Physics: Demonstrate understanding of mechanical systems – Angular Mechanics - Answers

In 2013, **AS 91524** replaced **AS 90521.**

The Mess that is NCEA Assessment Schedules….

In AS 90521 there was an Evidence column with the correct answer and Achieved, Merit and Excellence columns explaining the required level of performance to get that grade. Each part of the question (row in the Assessment Schedule) contributed a single grade in either Criteria 1 (Explain stuff) or Criteria 2 (Solve stuff). From 2003 to 2008, the NCEA shaded columns that were not relevant to that question.

In 91524, from **2013 onwards**, each part of a question contributes to the overall Grade Score Marking of the question and there are no longer separate criteria. There is no shading anymore. There is no spoon. At least their equation editor has stopped displaying random characters over the units.

For 2019:

NØ	N1	N2	A3	A4	M5	M6	E7	E8
0	1A	2A or 1M	3A or 1A +1M or 1E-	4 A or 2A + M or 2M or 1A+1E-	1A + 2M or 1M+1E- or 3A +1M or 2A + 1E-	2A + 2M or 3M or 3A + 1E- or 1A +1M + 1E-	2M+1E- or 2A +1M + 1E- or A + 2M + 1E-	A + 2M +E

Other combinations are also possible using a=1, m=2 and e=3. However, for M5 and M6, at least one Merit question needs to be correct (maximum 6). For E7 or E8, at least one Excellence needs to be correct (maximum 8). **Note: E- and E only applies to the E7 and E8 decision. <----** It's good that NCEA gets clearer and clearer after 16 years!

Question	Evidence	Achievement	Merit	Excellence
2019(2) (a)	Total $I = I_{MGR} + I_{children} = 271 + (3 \times 28.0 \times 2.10^2) = 641.44$ kg m^2	• Correct WORKING (Note, NOT answer as this is a SHOW question)		

(b)(i) (ii)	At max velocity Ek(rot) = ½ $I\omega^2$ = 388 J 388 = ½ 641 ω^2 ω_{max} = 1.10 rad s^{-1} $v = \omega r$ = 1.10 x 2.10 = 2.31 m s^{-1}	Correct working for ω_{max} = 1.1 rads^{-1} (Note, NOT answer as this is a SHOW question). OR Correct working for v = 2.31 m s^{-1}. (NB: NOT answer as this is a SHOW question.)	• All correct.	
(c)	$\alpha = \frac{\Delta\omega}{t} = \frac{0 - 1.10 \text{ rad s}^{-1}}{2.80 \text{ s}} = 0.393 \text{ rad s}^{-2}$ $\tau = I\alpha = 641.44 \text{ kg m}^2 \times 0.393 \text{ rad s}^{-2} = 252 \text{ N m}$	• α = 0.393 rad s^{-2} OR Calculates torque correctly with incorrect α value.	• All correct.	
(d)	• As the children move inward the mass distribution decreases, thus rotational inertia decreases. kg m^2 s^{-1} • States that angular momentum is conserved because it is a closed system or because the external torques sum to zero. • Assuming angular momentum is conserved a decrease in rotational inertia results in a proportional increase in angular velocity. • EK-rotational = ½ $I\omega^2$, even though I decreases proportionally, because ω is squared, the rotational kinetic energy increases overall. OR $E_{K\text{-rotational}} = \frac{1}{2} \times \frac{L^2}{I}$, I is reduced, so $E_{K\text{-rotational}}$ will increase.	• Recognises I decreases. OR Recognises that angular momentum is conserved so angular velocity increases. OR Recognises that angular velocity increases because mass is getting closer to the centre. *Accept inertia in place of rotational inertia.*	• Two concepts explained correctly with links. *Accept inertia in place of rotational inertia*	• All four concepts explained and linked. (E) • Three or more concepts explained and linked (including bullet point 4) but without the justification of the conservation of angular momentum. (E–) *Accept inertia in place of rotational inertia.*

2018(2) (a)	The period of rotation of the satellite must be the same as the period of the orbit. $T = 101 \text{ mins} \times 60 = 6060 \text{ s}$ $\omega = \frac{2\pi}{T} = 1.0368 \times 10^{-3} \text{ rad s}^{-1}$ $= 1.04 \times 10^{-3} \text{ rad s}^{-1} \text{ (3 sf)}$	Some evidence of working or reasoning leading to the correct answer. (This is a SHOW question)		
(b)	Pure rotational acceleration requires the torques to be additive but the forces to cancel. For this to occur there needs to be two thrusters whose forces cancel by acting in opposite directions but whose torques add. One thruster only would result in an unbalanced force and change the motion of the centre of mass – taking the satellite off course.	If there were only one thruster, it would deliver an unbalanced force. OR Pure rotation requires torque from equal and opposite forces.	If there were only one thruster, it would deliver an unbalanced force. AND Pure rotation requires torque from equal and opposite forces.	

(c)	$\omega_f = \omega_i + \alpha\Delta t$ $\omega_i = 0$ $\alpha = \frac{\omega_f}{\Delta t} = \frac{1.0368 \times 10^{-3}}{6.48 \times 10^{-3}} = 0.160 \text{ rad s}^{-2}$ (same answer if rounded value used) $\tau = F \times d$ There are two torques in the same direction. Total torque $\tau = 2 \times 5.00 \times 0.80 = 8.00 \text{ N m}$ $\tau = I\alpha$ $I = \frac{\tau}{\alpha} = \frac{8.00}{0.160}$ $I = 50.0 \text{ kg m}^2$ OR $\Delta p = F\Delta t$ $\Delta p = 5 \times 6.48 \times 10^{-3} = 0.0324$ N s for each thruster So total $\Delta p = 0.0648$ N s $\Delta L = \Delta p \cdot r$ $= 0.0648 \times 0.8$ $= 0.05184 \text{ kg m}^2 \text{ s}^{-1}$ $L = I\omega$ so $I = \frac{L}{\omega}$ $I = \frac{0.05184}{0.00104}$ $I = 49.9$ or 50 kg m^2	Correctly calculates angular acceleration = 160 rad s^{-2} OR Total torque = 8.00 N m OR Accept correct rotation calculation with incorrect torque of 16 Nm – used diameter rather than radius. OR Correctly calculates the total impulse (0.0648 N s or kgm s^{-1}) OR Correctly calculates the total change of angular momentum (0.05184 kg m^2 s^{-1}).	Correct answer supported by working.	

(d)	$I=\frac{1}{2}mr^2$ $=\frac{1}{2}\times 5.00\times 0.200^2=0.100\text{ kg m}^2$ $L=I\omega$ $L=50.0\times 1.0368\times 10^{-3}$ $L=0.0518\text{ kg m}^2\text{ rad s}^{-1}$ (if rounded value used: 0.520) $L_{\text{satellite}}=L_{\text{wheel}}$ $I_{\text{satellite}}\omega_{\text{satellite}}=I_{\text{wheel}}\omega_{\text{wheel}}$ $\omega=\frac{L_{\text{satellite}}}{I_{\text{wheel}}}$ $=\frac{0.0520}{0.100}=0.518\text{ rad s}^{-1}$ These calculations depend on the fact that there are no EXTERNAL torques acting on the satellite as angular momentum is conserved.	Calculates the rotational inertia of the flywheel/inertia-wheel. OR Calculates the angular momentum of the flywheel. OR Indicates recognition that that law of conservation of angular momentum will be involved. OR States that there are zero external torques.	Justifies use of the law of conservation of rotational momentum by stating that there are no external torques acting on the satellite. (Note: do not accept zero external forces) OR Correct answer for angular velocity of wheel.	Correct working with answer and states assumption that there are no external torques acting. (E) Correct working with answer and states an incomplete assumption. (E-)
2017(2) (a)	$\Sigma\tau=I\alpha$ $\Sigma\tau=58000\times 0.020$ $\Sigma\tau=1160$ So the torque produced by one rocket is 580 Nm.	Correct answer.		
(b)	$\omega_f^2=\omega_i^2+2\alpha\theta$ $\omega_f^2=0.58^2-2\times 0.020\times 2\pi$ $\omega_f^2=0.085$ $\omega_f=0.29\text{ rad s}^{-1}$	correct angle conversion. i.e. $\theta=2\pi$ OR Correct working except for angle conversion.	Correct calculation and answer.	

(c)	$\Sigma\tau = I\alpha$ As the rockets emit gas, the total mass of the spaceship will decrease. This will cause the rotational inertia to decrease. Torque is constant, so the angular acceleration will gradually increase.	States that decreased mass will cause rotational inertia (RI) to decrease. OR States that angular acceleration increases because RI decreases. *Accept inertia in place of rotational inertia.*	Decreased mass will cause rotational inertia to decrease. AND Angular acceleration increases because RI decreases. *Accept inertia in place of rotational inertia.*	
(d)(i) (ii)	The mass moves further away from the centre of rotation. This causes the rotational inertia to increase. There is no outside torque, so angular momentum is conserved. $L = I\omega$ so if the rotational inertia increases (changes), the angular speed will decrease (change), causing the period to increase (change). $L_i = L_f$ $58000 \times 0.45 = (58000 + 2740) \times \omega$ $\omega = 0.43 \text{ rad s}^{-1}$ $\omega = \frac{2\pi}{T}$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{0.43} = 14.6 \text{ (or 15) s}$	RI increase, so T increase. OR Angular momentum is conserved because there is no net external torque. OR Angular momentum is conserved so angular speed decreases. OR Correct angular speed. *Accept inertia in place of rotational inertia.*	Two concepts explained and linked. OR Calculated the period correctly from an incorrect angular velocity calculated using conservation of angular momentum formula (i.e. follow-on error carried forward) *Accept inertia in place of rotational inertia.*	All concepts explained and linked AND Correct calculation and correct answer for the period. (E) ### Correct calculation and correct answer for the period. (E-) OR All concepts explained and linked. (E-) *Accept inertia in place of rotational inertia.*
2016(2) (a)	Gravitational potential energy to both linear and rotational kinetic energy.	Correct answer.		

(b)	$\omega = \frac{v}{r} = \frac{0.250}{0.058} = 4.31 \text{ rad s}^{-1}$ $\text{RKE} = \frac{1}{2} I\omega^2$ $= \frac{1}{2} \times 0.14 \times 4.31^2$ $= 1.30 \text{ J}$	Correct angular speed OR Correct energy equation using incorrect angular speed.	Correct calculation and answer.	
(c)	One rotation $= 2\pi$ rad $\theta = \omega_i t + \frac{1}{2}\alpha t^2$ $2\pi = \frac{1}{2} \times 1.72 \times t^2$ $t^2 = \frac{2\pi}{\frac{1}{2} \times 1.72}$ $t = 2.70 \text{ s}$ OR $\omega_f^2 = \omega i^2 + 2\alpha\theta$ $\omega_f^2 = 0 + 2 \times 1.72 \times 2\pi$ $\omega_f = 4.65 \text{ rad s}^{-1}$ $\omega_f = \omega_i + \theta t$ $t = \frac{\omega_f - \omega_i}{\alpha}$ $= \frac{4.65 - 0}{1.72} = 2.70 \text{ s}$	Correctly converts rotation to radians. OR Correct calculation as evidenced by having SHOWN WORKING. OR Correct final angular velocity (ω_f = 4.65 rad s^{-1}). OR Allow *A* for follow on error for ω_f often giving t = 2.50 s.	Correct calculation and answer.	

(d)	The solid cylinder has a smaller rotational inertia since its mass is closer to centre compared to the hollow cylinder. Hence it will have a smaller proportion of rotational kinetic energy and a larger proportion of linear kinetic energy. (Since they both have the same amount of gravitational potential energy), the one with the greater proportion of linear kinetic energy will reach the bottom first. OR (Both cylinders will also have the same torque since they both have the same shape and mass.) The solid cylinder has its mass closer to the centre. So it has less rotational inertia so will have greater angular acceleration (since $\tau = I\alpha$). So will take less time to reach bottom.	One correct idea, e.g.: • Solid cylinder has less rotational inertia. • Or both have same torque. • Or solid cylinder has smaller "effective radius". • Or solid cylinder has mass closer to centre. (Accept converse arguments based on hollow cylinder also.) *Accept inertia in place of rotational inertia.*	Two key ideas linked, e.g.: Solid cylinder has less rotational inertia because mass is closer to centre. OR Solid cylinder has less rotational Inertia so will have less Rotational Kinetic energy (since $E_{KR} = ½ I\omega^2$) OR Solid cylinder has less rotational inertia so will have more linear kinetic energy. (Accept converse arguments based on hollow cylinder also.) *Accept inertia in place of rotational inertia.*	Complete correct succinct explanation. (E) Complete explanation with minor errors. (E–) *Accept inertia in place of rotational inertia.*
2015(3) (b)	$L = I\omega$ $I = \frac{L}{\omega} = \frac{3.24 \times 10^{-3}}{1.20} = 2.70 \times 10^{-3}\ \text{kg m}^2$	$I = 2.70 \times 10^{-3}$ kg m^2		
(c)	When the cat is in the air, no net external torque acts on it about its centre of mass, so the angular momentum about the cat's centre of mass cannot change. Since the front half of the cat has increased angular momentum, the rear of the cat must increase angular momentum by the same amount but in the opposite direction in order to maintain a total angular momentum of zero.	No external torques act. OR $L_{rear} = -L_{front}$	No external torques act so total angular momentum must be conserved so rear half of cat must rotate but in the opposite direction.	

(d)	$L_{rear} + L_{front} = 0$ so $L_{rear} = -L_{front} = 3.24 \times 10^{-3}$ If the falling cat pulls in its front legs, the cat can decrease its rotational inertia / moment of inertia by changing its mass distribution. If the falling cat stretches out its back legs, the cat can increase its rotational inertia / moment of inertia by changing its mass distribution. By changing its rotational inertia / moment of inertia, the cat can change the speed at which it rotates. Since the angular momentum of each half of its body is a constant. The cat rotates more quicky at the front of its body than at the back. $L_{rear} = -L_{front}$ $I_{rear}\,\omega_{rear} = I_{front}\,\omega_{front}$ Since $I \propto mr^2$, $I \propto r^2$ $\frac{\omega_{rear}}{\omega_{front}} = \frac{r^2_{front}}{r^2_{rear}}$ $\omega_{rear} = \frac{r^2_{front}\,\omega_{front}}{r^2_{rear}} = \frac{0.06^2 \times 1.20}{0.122} = 0.300 \text{ rad s}^{-1}$	$L_{rear} = 3.24 \times 10^{-3}$ kg m^2 s^{-1} OR The cat can change its mass distribution OR Explains that the cat can change the rotational inertia of the front and back parts of its body independently *Accept inertia in place of rotational inertia.*	Explains that the cat can change the rotational inertia of the front and back parts of its body independently. Tucking in the legs distributes more of the cat's mass close to the axis of rotation OR Stretching out the legs distributes more of the cat's mass further from the axis of rotation. Explains that the cat can change the speed at which different parts of its body rotate by changing the mass distribution. *Accept inertia in place of rotational inertia.* *Radius smaller is acceptable for mass closer to the centre/axis.* *For* ***(M)****, the candidate needs to have any two of the above three.*	• Explains that the cat can change the rotational inertia of the front and back parts of its body independently. Tucking in the legs distributes more of the cat's mass close to the axis of rotation so ω is quicker OR vice versa (i.e. stretching out the legs distributes more of the cat's mass further from the axis of rotation so ω is slower) Calculates $\omega_{rear} = 0.300$ rad s^{-1} *Accept inertia in place of rotational inertia.* *Radius smaller is acceptable for mass closer to the centre/axis.* *For* ***Excellence points (E),*** *the candidate needs to have all three bullet points.* *For* ***(E–)****, the candidate needs to have any two of the above three.*

2014(1) (a)(i)	$\omega = \frac{\Delta\theta}{\Delta t}$ $T = \frac{360°}{\omega} = \frac{360°}{14.7} = 24.49 \text{ days}$ $= 24.49 \times 24 \times 60 \times 60 = 2.11 \times 10^6 \text{ s}$	$\frac{360}{14.7} = 24.5 \text{ days}$ OR Correct ω: = 1.7014 × 10^{-4} ° s^{-1} OR ω = 2.96 × 10^{-6} rad s^{-1} (written with ω or with units OR $\frac{360}{14.7} \times 86\,400$ (shows subs but not equation) Correct working showing conversion of days to seconds.	Correct working showing conversion of days to seconds. AND Uses $\omega = \frac{\Delta\theta}{\Delta t}$ OR $T = \frac{1}{f}$ + proportional circle OR $T = \frac{360/2\pi}{\omega}$ OR Has described meaning of intermediate step, eg: 24.48 days for full rotation Or 1.7 × $10^{-4°}$ s^{-1} Or 2.96 × 10^{-6} rad s^{-1} Or show ratios, eg14.7 day 360 → 24.4 Or f = 4.73 × 10^{-8} Hz) *Can go backwards but has to be clear.*	

(a)(ii)	$v = \frac{2\pi r}{T} = \frac{2 \times \pi \times 6.96 \times 10^8}{2.12 \times 10^6} = 2.066.7$ $= 2070 \text{ m s}^{-1}$ *(Answer: 2060 – 2070 dependent on rounding.)*	Uses $v = \omega r$ with incorrect ω or T and correct r. OR Uses $v = \frac{2\pi r}{T}$ with incorrect T and correct r. OR Correct answer. OR Correct subs with wrong answer.	Correct speed 2060 – 2070 m s^{-1} with some working – (subs or eqn).	
(b)	When the core of the Sun collapses, this will cause the radius of the particles in the core to rotate with a smaller radius. Angular momentum will be conserved, so if the rotational inertia decreases, the core will have to rotate at a higher angular velocity.	Angular momentum is conserved. Rotational inertia of core will get smaller. **OR** Angular velocity increases because mass is closer to the centre / axis. *Accept inertia in place of rotational inertia*.	Angular momentum is conserved, therefore if rotational inertia decreases, angular velocity increases OR $L = I\omega$ therefore, if I decreases, ω increases. OR ω increases because I decreases due to mass closer to the centre / axis. *Accept inertia in place of rotational inertia.* *Radius smaller is acceptable for mass closer to the centre / axis.*	Angular momentum is conserved AND $L = I\omega$ AND I decreases due to mass closer to the centre / axis THEREFORE ω increases. *Accept inertia in place of rotational inertia.* *radius smaller is acceptable for mass closer to the centre / axis.*

(c)	For geostationary motion, the period of the satelite has to be equal to the period of Mercury And $F_c = F_g$ $\frac{GMm}{r^2} = \frac{mv^2}{r}$ $\quad v = \frac{2\pi r}{T}$ $r^3 = \frac{GMT^2}{4\pi^2}$ $r = \sqrt[3]{\frac{6.67\times10^{-11}\times3.30\times10^{23}\times(5.067\times10^{6})^2}{4\pi^2}}$ $r = 2.43\times10^{8}$ m	$F_c = F_g$ **OR** $\frac{mv^2}{r} = \frac{GMm}{r^2}$ **OR** $v = \sqrt{\frac{GM}{r}}$ $v = \frac{2\pi r}{T}$ Period of satelite period of Mercury *MAXIMUM 2As*	Merges ($F_c = F_g$ OR $\frac{mv^2}{r} = \frac{GMm}{r^2}$) And $v = \frac{2\pi r}{T}$, or $v = \omega r, \omega = 2\pi f$ with rearranging incorrect OR Using $F_c = F_g$ to derive $r = \frac{GM}{v^2}$ OR Full answer going backwards.	Correct rearrangement and substitution for r^3 or cube root of r AND ($F_c = F_g$ OR $\frac{mv^2}{r} = \frac{GMm}{r^2}$) AND $v = \frac{2\pi r}{T}$
(d)	<u>Interpreted as if Total probe:</u> The mass that is lost will have angular momentum, so the angular momentum of the space probe will decrease. The rotational inertia of the space probe will also decrease. The effect of these two changes will be that they cancel and the angular velocity will remain constant. <u>Interpreted as if Partial probe:</u> Angular momentum is conserved because of no external torque, therefore angular speed stays the same. / Instrument does not apply torque to the rest of the probe ,so angular speed does not change.	TOTAL probe: Loss of instrument means decrease in rotational inertia or angular momentum of probe OR PARTIAL probe: No torques therefore angular speed stays the same. OR MISINTERPRETATION: Idea that orbital speed is independent of mass of satellite. Has to show idea of mass cancelling from F_c or F_g or not in $v = \sqrt{\frac{GM}{r}}$	TOTAL probe: instrument takes L away and I away, therefore angular speed stays the same. $L = I\omega$ **OR** PARTIAL probe: instrument does not apply torque to the rest of the probe so the rest of the probe does not change angular speed.	

2013(1) (a)	SHOW THAT QUESTION $\omega = 2\pi f \quad = 2\pi \times 2.70 \quad = 16.965 \quad = 17 \text{ rad s}^{-1}$ OR $\omega = 2.70 \times 60 = 162$ rpm $162 \times \frac{2\pi}{60} = 17.0$	$2\pi \times 2.70$ OR working via 162 rpm		
(b)	SHOW THAT QUESTION $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{16.965}{0.250} = 67.86 = 68 \text{ rad s}^{-2}$ Or using $\omega_f = \omega_i + at$	Correct working – equation and substitution		
(c)	$\tau = Fr = 0.48 \times 0.034 = 0.01632$ Nm $\tau = I\alpha \Rightarrow I = \frac{0.01632}{67.86}$ $= 2.4050 \times 10^{-4} \text{ kg m}^2$ OR uses $I = 2/3\, mr^2$ Note: Using $L = mvr$ gets N	Correct τ. Uses equation $\tau = I\alpha$ correctly with any value of τ. Calculates an I using an mr^2 relationship, eg $I = mr$ (3.5836×10^{-4})	Correct answer. Unit not needed.	Both equations correct (or $2/3mr^2$ shown), with answer and unit (acceptable units: kg m^2, N m s^2, N m s^2 rad^{-1}, kg m^2 rad^{-1})

(d)(i)	A ball thrown with the same linear speed will reach the same height, 1.4m. The balls both have the same linear kinetic energy, which turns into the same amount of gravitational potential energy. As a result the balls both reach the same height of 1.4m. The rotation of the ball does not affect the height because the rotating ball stays rotating at the same angular velocity, so the rotational kinetic energy does not change, so the gravitational potential energy is not affected	Same height/ 1.4 m Non rotating ball goes to a lower height because there is less E_{Ktotal}, so there is less E_{Pgrav}.	Same height AND (linear kinetic energy is turned into gravitational potential energy OR Rotational kinetic energy does not change). The two balls have the same force/s/acceleration acting, so they reach the same height. OR The rotation does not affect the force/s/acceleration so they reach the same height.	Linear kinetic energy is turned into gravitational potential energy; they have the same linear kinetic energy, so they reach the same height. AND angular velocity/rotational kinetic energy doesn't change/affect the height The two balls have the same net force/ gravitational force acting, and the same initial speed, so they reach same height AND angular velocity/rotational kinetic energy doesn't change/affect the height
(d)(i)	Because a solid ball has a significant proportion of its mass closer to the centre of rotation it would have a smaller rotational inertia than the hollow ball. If both balls are given the same angular speed the solid ball needs less work to get it rotating than the hollow ball. If less work is done to get the ball rotating, more of the total work is done to give the ball linear velocity so it will have a greater release speed and so will rise higher because it has more kinetic energy that is changed to gravitational potential energy.	Solid ball has a smaller rotational inertia. Solid ball goes to a greater height. Note: Accept "inertia".	Smaller rotational inertia because the solid ball has mass closer to the centre. Less work done / energy to get the same spin of the solid ball OR more work / energy going into linear velocity / linear energy of the solid ball. (or less replaced with more IF they think I gets bigger) Less rotational kinetic energy because I is smaller (OR less replaced with more IF they think I gets bigger). Links linear kinetic energy to gravitational potential energy to height (even if height is incorrect). Note: Accept "inertia".	Links – solid ball has smaller I because mass is closer to centre of rotation therefore E_{Krot} is smaller – Work / E_{total} is the same for both therefore E_{Klin} is greater – E_{klin} turns into E_{Pgrav} - therefore the ball goes higher. Note: Accept "inertia".

2012(1) (a)	$\omega = 2\pi f \Rightarrow f = \frac{7.5}{2\pi}$ = 1.1937 rotations per second = 5.97 rotations in 5 seconds. OR $\theta = \omega t = 7.5 \times 5 = 37.5 \text{ rad} = \frac{37.5}{2\pi} = 5.97 \text{ rotations}$	[2]Correct answer.		
(b)	$\tau = I\alpha = I\frac{\Delta\omega}{\Delta t} = 5.45 \times \frac{7.50}{0.54} = 5.45 \times 13.89$ = 75.694 = 75.7 Nm	[2]Correct answer.		
(c)	Ira's legs are now further away from the centre of rotation than they were, so the rotational inertia has increased. This means she needs to apply a greater torque to achieve the same angular acceleration. As the push force is still being applied at the same place, and so the radius at which it is applied does not change, the force must increase to make the torque increase.	[1]Idea that greater rotational inertia means a greater torque is needed. (*Increase in rotational inertia explained can provide replacement evidence for 1(e)(i).*)	[1]Greater rotational inertia means a greater torque is needed. Angular acceleration must be the same.	[1]Correct and full answer.
(d)	Ira and the chair will rotate more slowly. If 'the system' is Ira, the chair and the book, no external torque is applied and so angular momentum is constant. The mass of the book increases the total mass of the system, so the rotational inertia increases and so ω decreases. When the book lands, friction between Ira's lap and the book (equal and opposite forces) will accelerate the book and cause a torque on Ira and the chair so that they slow down.	[1]Increase in rotational inertia stated and decrease in angular speed explained. OR Increase in rotational inertia explained.	[1]Increase in rotational inertia explained and decrease in angular speed explained.	
(e)(i)	As Ira pulls the book in, the rotational inertia of the system will decrease because she is shifting some of her mass closer to the centre of rotations. Because angular momentum must be conserved (no external torques acting), an increase in rotational inertia will cause a decrease in angular speed.	[1]Decrease in rotational inertia explained.	[1]*Increase in angular speed explained can provide replacement evidence for M1 in 1(d).*	

(ii)	$I_i = 5.45 + (2.1 \times 0.6^2) = 6.206$ $I_f = 5.45 + (2.1 \times 0.05^2) = 5.455$ $\omega_f = \frac{\omega_i I_i}{I_f} = \frac{5.00 \times 6.206}{5.455} = 5.69 \text{ rad s}^{-1}$ The movement of Ira's arms would further decrease *I*, so ω_f would be larger.	[1]Some idea of the rotational inertia of Ira's arm also affecting the final angular speed.	[2]Correct calculation of I_i or I_f	[2]Correct ω_f
2011(1) (a)	$f = \frac{4500}{60}$ $= 75 \text{ Hz}$ $\omega = 2\pi f$ $\omega = 2\pi \times 75$ $\omega = 471 \text{ rad s}^{-1}$	[2]Correct.		
(b)	$E_{k(rot)} = \frac{1}{2} I\omega^2$ $E_{k(rot)} = \frac{1}{2} \times 1.20 \times 10^{-3} \times 471^2$ $E_{k(rot)} = 133 \text{ J}$ $P = \frac{E_{k(rot)}}{t}$ $P = \frac{133}{0.12} = 1108 \text{ W}$	[2]Correct kinetic energy.	[2]Correct power.	
(c)	The electric motor exerts a force on the axle holding the blade. The wood exerts a force on the teeth of the blade. The torque from this force balances the torque from the motor. Because the force from the wood acts at a much greater radius, the size of this force is much smaller.	[1]Both forces described. OR One torque described. OR Torques on the blade stated to be balanced.	[1]Both torques described.	[1]Merit plus the torques are stated to be balanced.

2010(1) (a)(i)	$\omega = \frac{\theta}{t} = \frac{2\pi}{0.74} = 8.49 \text{ rad s}^{-1}$	[2]Correct answer		
(ii)	$E_K = \frac{1}{2}I\omega^2 = 0.5 \times 0.64 \times \left(\frac{2\pi}{0.74}\right)^2$ $= 23.1 \text{ J}$	[2]Correct answer		
(b)	Use the mass of the wheel as *m* and use an *r* value that is between the rim radius and the tyre radius. Assume the mass of the spokes and hub is negligible.	[1]Clear idea of how rotational inertia is affected by mass distribution.	[1]Mass distribution concept explained in terms of EITHER the need for a radius estimation OR the need to ignore the mass of both the spokes and the hub.	[1]Mass distribution concept explained in terms of both points.
(c)	Work done = Δ energy = E_K(lin) + E_K(rot) = ½mv^2 + 2 × ½$I\omega^2$, $v = r\omega \text{ so } \omega = \frac{v}{r} = \frac{5.5}{\left(0.724/2\right)} = 15.19$ $E_K = E_{K_{lin}} + E_{K_{rot}} = \frac{1}{2}mv^2 + 2 \times \frac{1}{2}m\omega^2$ $= \left(\frac{1}{2}68.4 \times 5.5^2\right) + \left(2 \times \frac{1}{2}0.64 \times 15.19^2\right)$ $= 1035 + 148 = 1183 \text{ J} = 1180 \text{ J (3 s.f.)}$	[2]Correct E_K(lin) OR [1]Correct concept of work = E_K(lin) + E_K(rot)	[2]Correct answer consistent with using the wrong radius or a diameter instead of a radius.	[2]Correct answer.

(d)	To accelerate the wheel a torque must be applied. As $\tau = I\alpha$, and as the mountain bike has less rotational inertia, less torque needs to be applied to accelerate it at the same rate. OR To accelerate the wheel it must be given rotational kinetic energy. As $E_K = ½I\omega^2$, and as the rotational inertia of the mountain bike is smaller, less energy will need to be given to it to accelerate it to a particular angular speed in a particular time.	[1]Idea that less torque must be applied to accelerate the mountain bike wheel. OR that less energy has to be supplied to give the mountain bike wheel its rotational kinetic energy.	[1]Concept given for achievement is clearly explained.	
2009(2) (a)	**THIS IS A SHOW QUESTION** $100 \times 2\pi / 60 = 10.5$ rad s^{-1}	[2]Correct working. $2\pi \times 1.66666$ OR 0.105×100		
(b)	$\omega_i = 0$ rad s^{-1} $\omega_f = 10.47$ rad s^{-1} $\theta = 1/3 \times 2\pi = 2\pi/3$ rad $\omega_f^2 = \omega_i^2 + 2\alpha\theta$ $\alpha = (\omega_f^2 - \omega_i^2)/2\theta = 10.5^2/(2\pi/3)$ $= 26.3$ rad s^{-2} $\tau = I\alpha$ $\tau = 16.5 \times 26.3 = 434$ N m	[2] Correct angle $\theta = 2\pi / 3$ rad = 2.09 rad	[2]Correct α. $\alpha = 26.3$ rad s^{-2}	[2]Correct answer. $\tau = 434$ N m *OR* $\tau = 432$ N m

(c)	For the plane to be easier to start Sam would need to apply less force / less torque for the same α or do less work. For less force he would need less torque $\tau = Fr$ $\tau = I\alpha$ so for less torque with same α, he must reduce I. For less work he needs to produce less gain in E_k for the same ω. $E_K = \frac{1}{2}I\omega^2$, so again I must be reduced. Either way he has to reduce I for the propeller. $E_K = \frac{1}{2}I\omega^2$ So to reduce I, he could reduce mass by making the whole propeller thinner, or make a new one out of a less dense material. It could be re-shaped so that more of the mass was close to the axis.	[1]Rotational inertia is identified as the key concept.	[1]The explanation explains how a lower moment of inertia is achieved by changing: **radius** (mass closer to axis) **OR** **mass** e.g. thinner less dense lighter	[1]Explanation covers why the propeller is easier to accelerate. Easy to accelerate means less Torque Less Torque means less I $\downarrow\tau = \downarrow I\, a$ **AND** Need to explain how a lower moment of inertia is achieved by changing: **radius** (mass closer to axis) **AND** **mass** e.g. thinner less dense lighter
(d)	$\omega_i = 37.7$ rad s^{-1} $\omega_f = 0$ rad s^{-1} $t = 20$ s $\theta = ?$ $\theta = (\omega_f + \omega_i).\ t/2 = (0 + 37.7)20/2 = 377$ rad $377/2\pi = 60$ revolutions Assume constant angular acceleration / constant torque.	[2]$\theta = 377$ rad or $\theta = 378$ rad [1]Correct statement. ***a* constant** **τ constant**	[2]Correct answer. **60** revolutions	

2008(1) (a)	$\tau = Fr$ = 132 N × 0.83 m = 109.56 = 110 Nm (2sf)	[2]**SHOW** question. Correct working shown.		
(b)	$\tau = I\alpha \quad I = \frac{\tau}{\alpha} = \frac{110}{0.48} = 229 = 230$ kg m^2 If you use 109.56 you get 228.25.	[2]**SHOW** question. Correct working.		
(c)	final $E_{K(ROT)} = \frac{1}{2} I\omega^2$ = ½ × 230 × 0.58^2 = 39 J (2 sf) OR 38 J (2sf) (109.56 × 0.35) = 38.39 J)	[2]Correct numerical answer for E_k. [1] Correct **sf and units**	If uses work = 109.56 × 0.35 OR uses *I* = 228.25 then rounding will be 38 J Otherwise 39 J	

(d)	The **force** moved a **distance**: so **work** (*Fd*) was done that **transferred energy** to the door. (assuming F constant) OR the force caused a **torque** ($\tau = Fr$) which moved through an **angle** so **work** ($\tau\theta$) was done and **transferred energy.** **(1) Links force to work in context.** $F \rightarrow \tau \rightarrow work^{(a)} \rightarrow \Delta KE$ $F \rightarrow work^{(b)} \rightarrow \Delta KE$ (a) work could be torque and rotation / angle / tuning (b) work is force and distance moved. **(2) Links work to change in energy.** Work causes the (change in) energy (kinetic energy). **(3) Links force to acceleration and then work.** $F \rightarrow \tau \rightarrow \alpha \rightarrow \Delta\omega \rightarrow (\Delta)KE$ Argument must be a rotational one (not linear).		[1]Any of (1) OR (2) OR (3) If have not explained the gain in KE of the door OR explanation (3) is almost complete.	[1]BOTH of (1) AND (2) OR (3) They must explain the **gain in KE** for Excellence

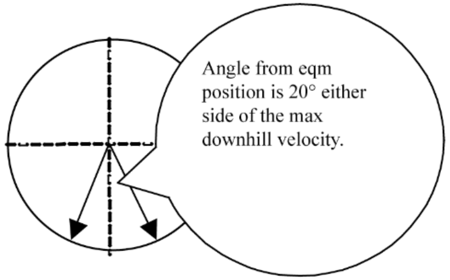
(e)	Yes because: If torque is the same, as $\tau = Fr$, if *r* is bigger, ***F* is smaller** The reason why torque must be the same is because this causes - same time and same velocity - or the same angular acceleration.		[1]Torque constant and therefore as radius (r) increases F decreases	[1]Justifies why the torque must be constant, e.g.: Constant τ gives angular acceleration. OR Constant τ gives same time and same velocity
(f)	Angle travelled during acceleration $\omega_f^2 = \omega_i^2 + 2\alpha\theta \quad \omega_i = 0$ $\frac{\omega_f^2}{2\alpha} = \theta = \frac{0.58^2}{2 \times 0.48} = 0.35$ radian so after she stops pushing angle is 2.0 – 0.35 = **1.65 rad.**	[2]Correct calculation of angle travelled during acceleration. (0.35 rad)	[2]Correct answer.	
2008(3) (b)	$\omega = 2\ f = \frac{2}{t} = \frac{2}{0.91} = 2.16 \text{ rad s}^{-1}$ $V_{max} = r\omega$ $= 0.69 \times 2.16 = 1.49 \text{ m s}^{-1}$ Max speed is when the swing and the whole chair are going up: *v*= 1.5 + 1.2 = 2.7 m s^{-1}	[2]ω correctly calculated OR Adds their (incorrect) V_{max} to 1.2ms^{-1}	[2]V_{max} =1.49 ms^{-1} OR Consistent V_{max} calculated from their incorrect ω	[2]Correct answer 2.7 ms^{-1}

<table>
<tr>
<td>(c)</td>
<td>
The chair will be travelling at 0.2 m s^{-1} downhill velocity when its velocity relative to the equilibrium position is

1.2 + 0.2 m s^{-1} down the hill.

i.e. when v is 1.4 m s^{-1} to the right relative to the pivot.

$v = A\omega \cos \omega t$
$1.4 = 0.69 \times 2.16 \times \cos \omega t$
$\omega t = \cos^{-1}\left(\frac{1.4}{0.69 \times 2.16}\right) = 0.35 \text{ rad} = 20^{\circ} \quad \text{OR} \quad 21^{\circ}$

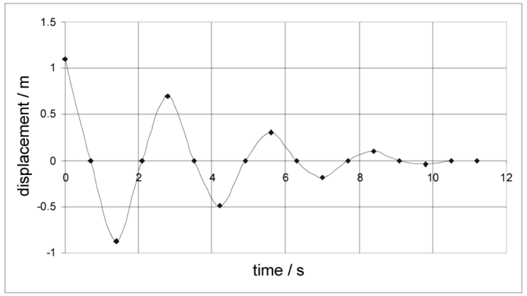


<table>
<tr><td></td><td></td><td colspan="2">V_{max} SHM</td></tr>
<tr><td></td><td></td><td>1.5
or uses cos formula</td><td>2.69</td></tr>
<tr><td rowspan="4">V_{max}
Linear</td><td>1.0</td><td>48°
.78 rads
t = 0.361 s</td><td>68°
1.19 rads
t = 0.551 s</td></tr>
<tr><td>1.2</td><td>37°
0.64 rads
t = 0.296 s</td><td>64°
1.11 rads
t = 0.514 s</td></tr>
<tr><td>0.2</td><td>82°
1.43 rads
t = 0.662 s</td><td>86°
1.50 rads
t = 0.695 s</td></tr>
<tr><td>1.4</td><td>E2
21°
0.37 rads
t = 0.162 s</td><td>59°
1.0 2rads
t = 0.472 s</td></tr>
</table>
</td>
<td>[2]Correct calculation of v relative to the pivot. (1.4 m s^{-1})</td>
<td>[2]Consistent answer from an incorrect V_{max}

OR

uses sin (instead of cos) to get 70°

(1.22 or 1.21 rads)</td>
<td>[2]Correct answer of 20° or 21°.</td>
</tr>
</table>

(d)		[1]Graph shows amplitude decreasing over 4 oscillations.	[1]Achieved and starts the graph at correct position AND shows period is the same.	
2008(4) (d)	As he starts to rotate the club, he will rotate in the opposite direction (the total angular momentum will remain constant). As he hits the ball, he will recoil (total linear momentum will remain constant). Watch for incorrect directions **(N)**	[1]Rotation or Translation		
2007(1) (a)	There is more mass on one side of the centre of curvature than the other.	[1]Idea of uneven spread of mass.		
(b)	The bowl is in contact with the surface at point P. The gravity force does not act through point P and so will create a clockwise torque on the bowl that will make it roll over to the right.	[1]Recognition that a torque is needed / that the forces are not in a line.	[1]Correct explanation of how a torque on the bowl is created.	
(c)	$E_K(\text{rot}) = \frac{1}{2}I\omega^2$ $= \frac{1}{2} \times 2.16\times10^{-3} \times 2.8^2$ $= 0.0084672$	[2]Correct working.		

(d)	E_K(lin) =½mv^2 $v = r\omega$ = 0.060 × 2.8 = 0.168 m s^{-1} E_K(lin) = ½ × 1.50 × 0.168^2 = 0.021168		[2]Correct working.	
(e)	From the gravitational potential energy lost by the bowl as its centre of mass falls.	[1]Correct idea of gravitational potential energy.		
(f)	$mgh = E_K(\text{rot}) + E_K(\text{lin})$ $\Rightarrow h = \dfrac{0.0084672 + 0.0211}{1.50 \times 9.81}$ = 2.0139×10^{-3} = **2.0 × 10^{-3} m**		[2]Correct answer.	
(g)	The unbalanced torque on the bowl will try to make it rotate. This will cause the bottom bit that is in contact with the flat surface to try to slip backwards to the left. Friction acting against this motion must therefore act to the right.	[1]Some idea of friction opposing the turning of the bowl.	[1]Correct idea of friction opposing the turning motion of the bowl.	[1]Correct idea of friction acting against the backwards push of the turning bowl on the surface.
(h)	Because the frictional force acts on the moving bowl at right angles to its motion, it will act as a centripetal force and will cause the bowl to move in a circular path.	[1]Idea that a force that acts at an angle is needed.	[1]Idea of friction providing the centripetal force.	[1]Correct idea of friction being at an angle to the direction of motion and therefore providing the centripetal force.
2006(3) (a)	$I = \frac{2}{5}mr^2 \Rightarrow 5 \times 3.73 = 2 \times 76 \times r^2$ $\Rightarrow r^2 = 0.122698 \Rightarrow r = 0.35$ m	[2]Correct answer.		

(b)	When Hopi tucks his body, his mass becomes concentrated closer to his axis of rotation, so reducing his rotational inertia. Angular momentum will be conserved and so his angular speed will increase. As the time Hopi has to execute the dive is fixed, rotating at a faster speed enables him to complete all the rotations.	[1] Idea that change in mass distribution is the key factor for decreasing I.	[1] Explanation shows the link between changing mass distribution and reducing I leading to increasing ω.	[1] Explanation shows the link between changing mass distribution and reducing I leading to increasing ω from conservation of L and makes reference to the limited time available for the dive.
(c)	$L = I\omega = 3.73 \times 9.82 = 36.6286$	[2] Correct working.		
(d)	$\omega = \frac{\theta}{t}$, $\theta = 2 \times 2\pi$ radians $t = \frac{2 \times 2\pi}{9.82} =$ $= 1.2797 = 1.28$ s		[2] Correct answer.	
(e)	Conservation of angular momentum.	[1] Correct answer.		

(f)	$I\omega$ (initial) = $I\omega$ (final) I (final) = 5 × I (initial) $\frac{I_f}{I_i} = \frac{\omega_i}{\omega_f} = 5$ $\Rightarrow \omega_f = \frac{\omega_i}{5} = \frac{9.82}{5} = 1.964$ $\omega_f = \omega_i + \alpha t$ $\Rightarrow \alpha = \frac{(1.964 - 9.82)}{0.32} = -24.55$ angular deceleration = 25 rad s^{-2} $I_f = 18.65$	[1]Recognition that conservation of angular momentum can be used to find the final angular speed.	[2]Correct final angular speed.	[2]Correct answer.
2005(1) (b)	$v = r\omega \quad \Rightarrow \omega = 0.26 \div 68$ $= 3.82353 \times 10^{-3}$ rad s^{-1}	Correct answer		
(c)	$T = \frac{1}{f}$, $\omega = 2\pi f \quad \Rightarrow T = \frac{2\pi}{\omega}$ $= \frac{2\pi}{3.82353 \times 10^{-3}} = 1643.29$ = **1600 s or 27 minutes**		Correct answer	
(d)	$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{3.82353 \times 10^{-3}}{2.3}$ $= 1.66240 \times 10^{-3}$ = $\mathbf{1.7 \times 10^{-3}}$ **rad** $\mathbf{s^{-2}}$	Correct answer		

(e)	$\theta = \frac{(\omega_i + \omega_f)}{2}t$ $= \frac{3.82353 \times 10^{-3}}{2} \times 2.3$ $= 4.39706 \times 10^{-3}$ rad $= 180 \div \pi \times 4.39706 \times 10^{-3}$ $= 0.25193$ **0.25°**		Correct answer. Accept any valid method.	
(f)	$\tau = I\alpha$ $\Rightarrow I = 46 \times 10^{6} \div 1.66240 \times 10^{-3}$ $= 2.76708 \times 10^{10}$ $\mathbf{2.8 \times 10^{10}}$ **kg m²**	Correct answer.		
(g)	As the capsules increase speed, the frictional forces acting against their motion also increase, causing the unbalanced torque and hence angular acceleration to decrease. When the torque of these frictional forces becomes equal to the applied torque, the torques will be balanced and so the wheel will no longer be accelerating.	[1]One correct and relevant statement: frictional torques act against the motion / at constant speed the torques are balanced.	[1]Explanation shows recognition that, even though the wheel is travelling at constant speed, a torque must still be applied to balance the torque of frictional forces.	[1]*Explanation includes the idea of* ***increasing*** *frictional torque.*
(h)	At the rim of the wheel.	Idea that the point should be as far from the centre of rotation as possible.		
(i)	The wheel is travelling at constant speed, so there is no change in kinetic energy. The gain and loss in gravitational potential energy are the same, so there is no net change. The only energy transformation produces heat due to all the frictional effects that are acting against the motion of the wheel.	One correct and relevant statement: friction converts energy to heat / no change in kinetic energy / no change in gravitational potential energy.	Recognition that the constant speed of the wheel means there is no change in kinetic energy / Gain and loss in gravitational potential energy of the capsules is balanced out. / Only energy change is to heat by friction.	No change in kinetic and gravitational potential energies fully and clearly explained.
(j)(i)	$\theta = \omega t$ $= 3.82353 \times 10^{-3} \times 29.0$ $= 0.110882$ **0.11 rad (6.4°)**	Correct answer.		

(j)(ii)	$2 \times d = r\theta = 68 \times 0.110882$ $= 7.54 \quad \Rightarrow d = 3.77$ **3.8 m**		Correct answer. (Accept if d = vt is used.)	
(k)	Angular momentum is not conserved because L = Iω. Rotational inertia has increased because the total mass has increased but the angular speed, w, stays constant because the speed of the capsules is constant.	One correct and relevant statement: change in mass causes change in angular momentum / rotational inertia increases / angular speed stays constant.	Explanation clearly links changing mass to a change in angular momentum of each capsule. I expressed in terms of angular velocity.	Explanation is clear, concise and complete. Correct relationship derived.
(l)	$E_{K(ROT)} = \frac{1}{2}I\omega^2 \quad I = 2 \times E_{K(ROT)} \div \omega^2$ $E_{K(ROT)} = E_{K(LIN)} = \frac{1}{2}mv^2$ $\Rightarrow I = 2 \times \frac{1}{2}mv^2 \div \frac{v^2}{r^2} \quad = mv^2 \times \frac{r^2}{v^2}$ $= mr^2$	Recognition that $1/2I\omega^2$ can be equated to $1/2mv^2$	I expressed in terms of angular velocity.	Correct relationship derived.
(m)	L = mvr = 65 x 0.26 x 68 = 1149.2 = 1100 kg m^2 s^{-1}	Correct answer. (Accept if L = Iω is used.)		