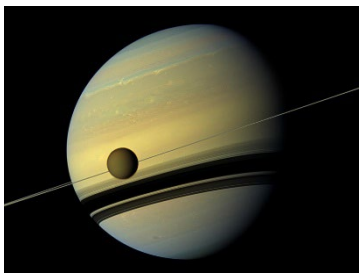
MECHANICS: LINEAR MECHANICS QUESTIONS

MOMENTUM AND CIRCULAR MOTION (2025;1)



Titan is the largest of Saturn's moons. For the sake of this question, you can ignore the gravitational effects of the other moons of Saturn and the rest of the objects in the solar system and assume that Titan is in a perfectly circular orbit.

Mass of Saturn: 5.68×10^{26} kg
Radius of Saturn: 5.82×10^{7} m Mass of Titan: 1.35×10^{23} kg
Distance between the centres of mass of Saturn and Titan: 1.22×10^{9} m

(a) Show that the centre of mass of Saturn and Titan is 2.90×10^{5} m from the centre of Saturn.

(b) Two ice particles around Saturn move toward each other, as shown below. Particle A has a mass of 2.45 kg and a velocity of 8.00 m s^{-1}; particle B has a mass of 3.20 kg and a velocity of 15.0 m s^{-1}. The particles collide and stick together without losing any mass.

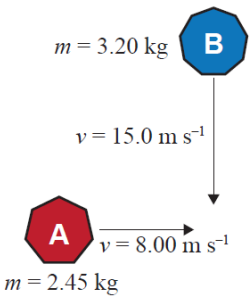
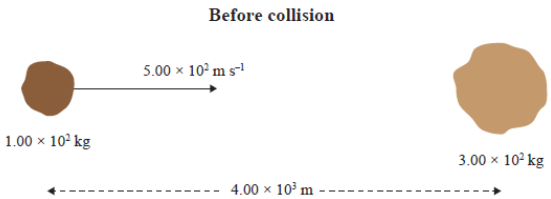


Diagram:

Calculate the size and direction of the momentum of the two particles when they are stuck together. Determine the direction relative to the initial velocity of particle A. *A diagram may assist your answer.*

COLLISIONS IN SPACE (2022;2)



Comets and asteroids are loosely formed objects that are easily broken up, especially when they collide. In this scenario a 1.00 x 10^2 kg object moving with a velocity of 5.00 x 10^2 m s^{-1} collides with a stationary, 3.00 x 10^2 kg object.

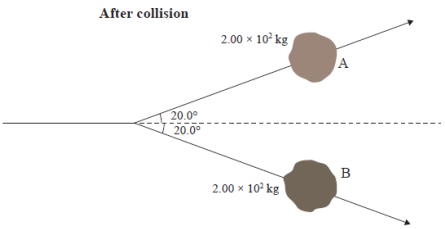
(a) Calculate the distance of the centre of mass of the system from the 3.00 x 10^2 kg object when the objects are 4.00 x 10^3 m apart.

(b)

(i) Calculate the gravitational force between the two objects, when their centres of mass are 10.0 m apart. Assume that the velocities of the masses are constant before the collision.

(ii) Hence find the acceleration of the 1.00 x 10^2 kg object at this separation, to determine whether the assumption that their velocities are constant before the collision is valid.

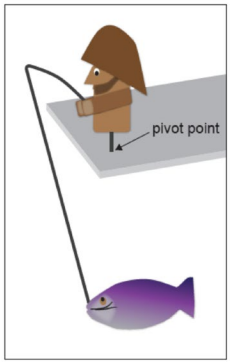
(c) The objects collide, with no external forces, and form two 'rocks', A and B, each with a mass of 2.00 x 10^2 kg. These move away from each other, as shown in the diagram, in directions at 20° either side of the initial direction of the impacting rock. Show that the size of the momentum of rocks A and B are the same, even though they are in different directions. You may want to draw vector diagrams.

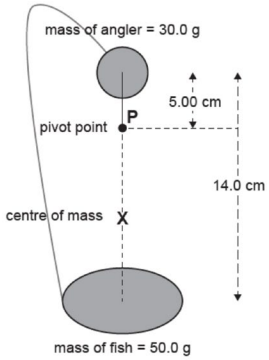


(d) Calculate the speed of rocks A and B. You may want to draw vector diagrams. Begin your answer by calculating the total momentum before collision.

FISHING TOY (2021;1)

A simple balancing toy can be made by joining two masses with a rigid wire. The 'Balancing Angler' toy below is an example. It can be approximated to two masses, joined by wire, as shown in the diagram. The distance between the centres of mass of the angler and the fish is 14.0 cm. The pivot point is at the bottom of the pole under the angler and is 5.00 cm below the centre of mass of the angler.





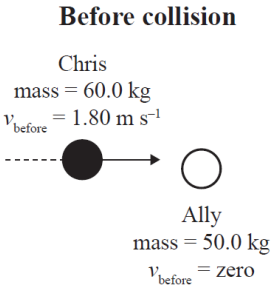
(a) Assuming that the wire has no mass, show that the centre of mass (X) of the system is 3.75 cm (3.75 x 10^{-2} m) below the pivot point (P), along the line joining the angler and the fish.

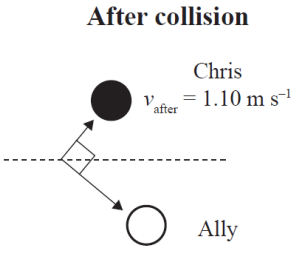
QUESTION ONE (2019;1)

Ally and Chris are rollerblading. Assuming friction is negligible, the system of Ally and Chris can be considered an isolated system in the horizontal direction.

(a) State a relevant physical quantity that is conserved during a collision between Chris and Ally.

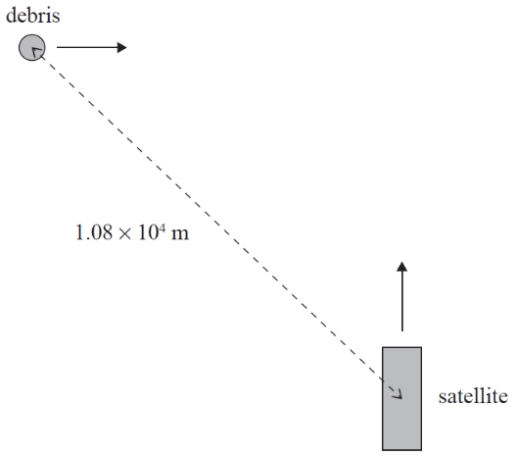
(b) At one instant, Ally stops, and Chris collides with her. They move off at right angles to each other, as shown in the diagram below. Show that Ally's speed after the collision is 1.71 m s^{-1}.





QUESTION ONE (2018;1)

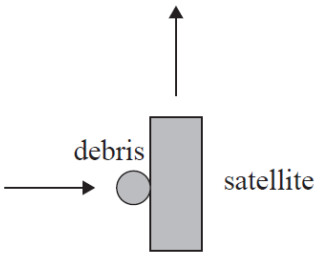
The Electron Rocket developed by New Zealand company Rocket Lab has begun commercial launches of satellites from the Mahia Peninsula in Hawke's Bay. The rocket can carry a satellite of mass 1.50×10^2 kg to a stable, circular orbit 5.00×10^5 m above the Earth's surface.



20.0 kg piece of space debris is travelling at the same altitude and speed as the satellite but in a direction that is 90° to the satellite's velocity. At the moment shown in the diagram, the debris and the satellite are 1.08×10^4 m apart. The satellite is travelling at 7.61×10^3 m s^{-1}.

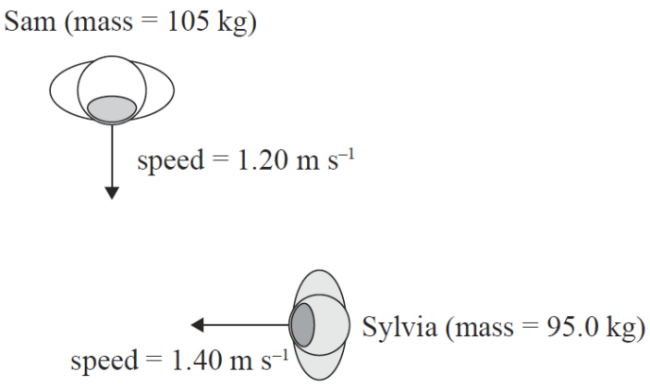
(c) Calculate the distance between the centre of mass of the system and the satellite. Describe the motion of the centre of mass of the system as the debris and satellite continue to move along their paths. Justify your response.

(d) The debris collides and becomes embedded in the satellite. Calculate the speed and direction (relative to the initial direction of the satellite) of the satellite and embedded debris.



QUESTION ONE (2017;1)

Two astronauts, Sylvia and Sam, are on a mission to another planet. During their journey they are doing a "spacewalk" outside their spaceship. At one time they are moving freely as shown in the diagram below. They collide and stick together.



(a) Calculate the distance between Sam and the centre of mass of the system when he and Sylvia are 4.80 m apart.

(b) Describe what happens to the centre of mass of the system as the astronauts move closer together and then collide.

(c) Calculate the astronauts' combined speed after they collide.

CATS AND GRAVITY (2015;3)

Cats have the ability to orient themselves in a fall, allowing them to avoid many injuries even when dropped upside down. Cats can do this even without tails to help them and they do not need to be rotating first.

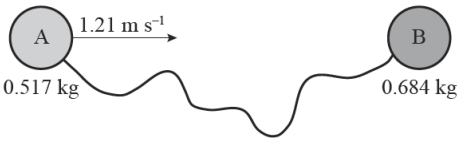
The sequence of events for a typical 3.00 kg cat:

- The cat determines which way is up (by rotating its head).
- The cat exerts internal forces to twist the front half of its body to face down (by twisting its spine around its centre of mass and aligning its rear legs).
- Then the cat exerts internal forces to twist the back half of its body to face down (by arching its back).
- The cat lands safely.

The cat can be modelled as a pair of equal mass cylinders (the front and back halves of the cat) linked at the centre of mass of the cat. The moment of inertia, $I \propto mr^2$.

(a) Describe the motion of the centre of mass of the cat during its fall and explain why the linear momentum of the cat is increasing.

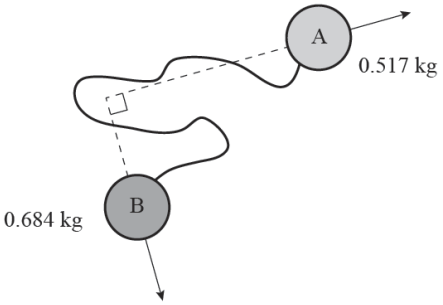
TRANSLATIONAL MOTION (2014;3)



A system consists of two discs, A and B, attached together with a light cord. The discs slide across a frictionless surface. Disc A has mass 0.517 kg and disc B has mass 0.684 kg. Disc B is stationary, and disc A is moving towards disc B with a speed of 1.21 m s^{-1}.

(a) Show that the speed of the centre of mass of the system is 0.521 m s^{-1}. Show all your working.

(b) The discs collide and after the collision they are moving at right angles to each other. Disc A receives an impulse of 0.250 N s.



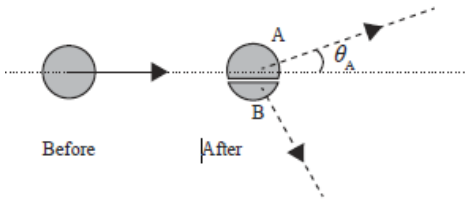
(i) Show that the speed of disc B after the collision is 0.365 m s^{-1}. Explain your reasoning.

(ii) Determine the size of the momentum of disc A after the collision.

(c) The discs continue to slide until the cord is fully extended. When this happens, both discs change their speed and direction. By considering the force(s) that act on the discs, explain why the momentum of the system must be conserved.

LINEAR MOTION (2012;3)

A special effect for a movie requires a ball to 'explode' in mid-air after it has been thrown. This is achieved by using a strong spring to push the two pieces of the ball apart. The ball has a mass of 0.105 kg and, at the instant it splits, it is travelling horizontally at 67.0 m s^{-1}. A fraction of a second after the split, A and B are effectively still travelling horizontally in the directions shown. Piece A has a mass of 0.0800 kg and is travelling at 80.0 m s^{-1}. The only external force on the ball is gravity: there are no external horizontal forces.



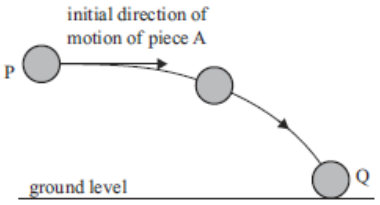
The ball and ball pieces, viewed from above.

(a) Discuss whether momentum and/or kinetic energy is conserved during the explosion.

(b) Calculate the horizontal momentum of the system of two pieces immediately after the explosion.

(c) Piece A has a mass of 0.0800 kg and is travelling at 80.0 m s^{-1}, at an angle of θ_A, as shown. By drawing a vector diagram, show how you could calculate the velocity of piece B immediately after the explosion. (A calculation of the velocity of B is not required.)

Now consider the effect of gravity as the two pieces fall to the ground. A side view of the path of piece A is shown in the diagram below. As piece A falls from P to Q, the vertical component of its momentum increases by 3.1 kg m s^{-1}.

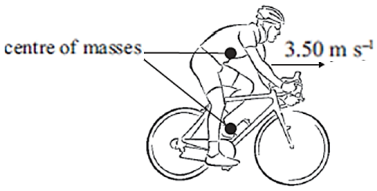


(d) Discuss the impulse on piece A as it falls from P to Q, explaining the cause of the impulse and the effect that the impulse has on the momentum of A.

(e) Calculate the velocity of A at the instant it hits the ground.

LINEAR MOTION (2010;2)

A cyclist, of mass 55.0 kg, is riding a bike of mass 11.0 kg. The bike and cyclist can be considered a system. While freewheeling (not pedalling) at constant speed of 3.50 ms^{-1}, the cyclist positions himself so that his centre of mass is vertically above the centre of mass of the bike. He then moves his position so that his centre of mass moves towards the front of the bike.



(a) State why momentum will be conserved during this change in position.

(b) Relative to the bike, the cyclist moves his centre of mass forward a horizontal distance of 13.2 cm at a steady speed. Show that the horizontal distance between the centre of mass of the system and the centre of mass of the bike is now 11.0 cm.

The cyclist goes over a steep drop and crashes on the ground. When the bike (with cyclist) hits the ground it is travelling horizontally at 3.50 m s^{-1} and vertically at 1.80 m s^{-1}. It takes 0.835 s, from the first moment of impact, for the bike (and cyclist) to stop.

(c) Calculate the speed of the bike as it hits the ground.

(d) Calculate the size of the average total stopping force which acts on the bike and cyclist.

(e) During the crash, the bike (and cyclist) experience an upwards push from the ground that stops the vertical motion, and friction with the ground that stops the horizontal motion. Horizontal motion stops after 0.835 s but the vertical motion stops in about one tenth of this time. Describe and explain how the vertical and horizontal forces on the bike (and cyclist) will be different.

(f) Cycle helmets are made so that they will crumple under impact. Explain why the crumpling of the helmet reduces the risk of serious injury to the cyclist.

QUESTION FOUR (2009;4)

After flying, Sam relaxes with a game of pool. The diagrams below show what appeared to occur during one shot. Ball 1 has a mass of 0.146 kg and ball 2 has a mass of 0.165 kg. Before the collision Ball 1 has a velocity of 9.00 ms^{-1} to the direction shown and a momentum of 1.31 $kgms^{-1}$. After the collision, Ball 1 has a velocity of 4.00 ms^{-1} in the direction shown.

(a) Calculate the velocity of the centre of mass of the system before the balls collide. You can assume that the diameter of the balls is small so that ball 1 is travelling directly towards ball 2.

(b) State what happens to the velocity of the centre of mass of the system during the collision. Give a reason for your answer.

(c) Calculate the size of the change in velocity of ball 1.

(d) Calculate the final velocity (magnitude and direction) of ball 2.

(e) By using $E_k = ½mv^2$, Sam finds that the total linear kinetic energy of the two balls increases during the collision. By considering all possible forms of energy involved, provide a reason why this collision is theoretically possible.

CENTRE OF MASS (2008;2)

Many ski resorts provide chairlifts to carry skiers to the top of the mountain. The chairs hang from a single suspension point on a steel cable. Figure A shows a front view of an empty chair. (Note that the term chair refers to the whole frame, from the pivot down, and the seat.) The whole chair is rigid. It hangs freely from a pivot point on the cable.

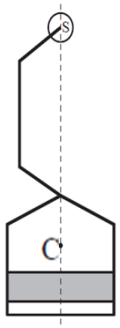


Figure A

(a) An empty chair hangs with the centre of mass (C) vertically below the pivot point (S). Draw vector arrows to represent the two forces that act on the chair. Add labels naming these forces.

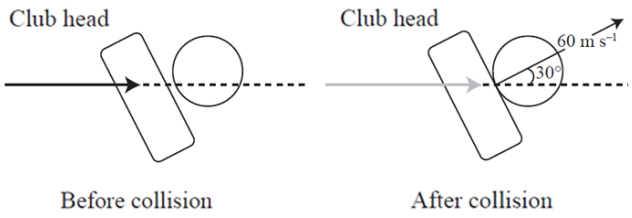
(b) A heavy skier sits in the chair at X (Figure B). Explain why the chair moves and why it hangs in this new equilibrium position.

Figure B

GOLF IN SPACE (2008;4)

In November 2006, flight engineer Mikhail Tyurin hit a golf ball while he was in space, orbiting Earth on a mission on the International Space Station.

(c) Consider the golf shot as a collision between a club head, of mass 0.20 kg, and the ball. Velocities are measured relative to the orbiting space craft. The ball (mass 3.0 x 10^{-3} kg) is initially stationary. After being hit, it has a velocity of 60 ms^{-1}.

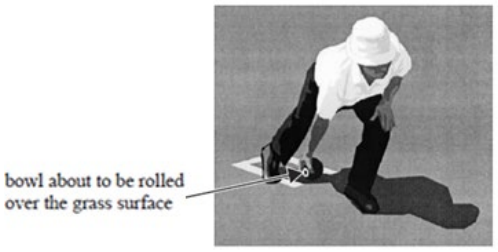


(i) Calculate the momentum lost by the club head during the collision and show the direction of this lost momentum on the 'After collision' diagram.

(ii) The initial velocity of the club head is 40 ms^{-1} horizontally. Analyse the collision using momentum vectors to show that the velocity of the club head is virtually unchanged by the collision.

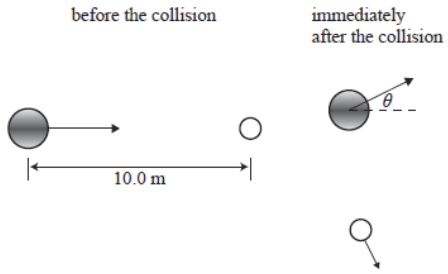
(iii) Explain how it is possible for the ball to leave the club head faster than the initial speed.

QUESTION ONE (2007;1)



In lawn bowls, players roll the bowls over a horizontal grass surface towards a small target ball called a jack. The aim of the game is to get more of your bowls closer to the jack than those of your opponents.

When the bowl is rolled at high speed, the path of the bowl can be considered to be straight. Consider the bowl hitting the stationary jack so that, immediately after the collision, the jack and the bowl are travelling at right angles to each other. The bowl has mass 1.50 kg; the jack has mass 0.25 kg.



The speed of the centre of mass of the system of bowl and jack is 5.4 m s^{-1} (assume this speed is constant).

(a) Show that the momentum of the system of bowl and jack is 9.5 kg m s^{-1}.

(b) State why the momentum of the bowl before the collision is 9.5 kg m s^{-1}.

In the diagram above, the position of the moving bowl, before the collision, is 10.0 m from the jack.

(c) Show that the distance of the centre of mass of the system of bowl and jack from the jack is 8.57 m.

(d) Calculate the time it takes the bowl to travel from the position shown to the jack.

The momentum of the bowl after the collision is 8.7 kg m s^{-1}.

(e) Calculate the angle, θ (in the diagram above), through which the bowl turns during the collision.

(f) During the collision, the bowl and jack are in contact for 0.025 s. Calculate the size of the force exerted on the bowl.