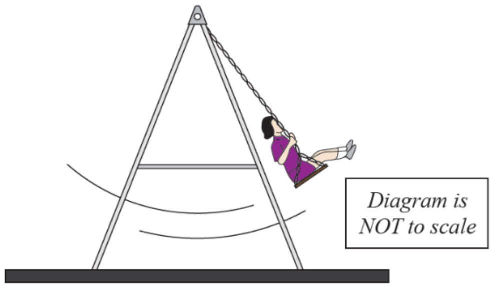
MECHANICS: SIMPLE HARMONIC MOTION QUESTIONS

QUESTION THREE (2019;3)

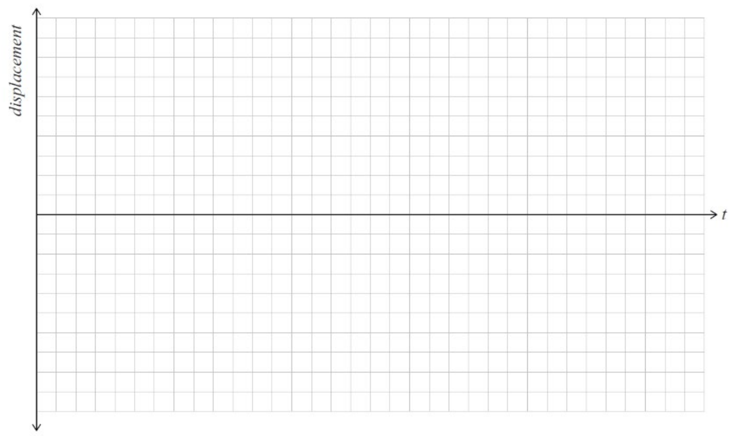
Jay is enjoying a swing at the playground. The period of one oscillation is 2.40 s and Jay maintains a constant amplitude of 0.310 m by swinging her legs back and forth to replace the energy lost due to friction. The mass of the system (Jay and the swing) is 70.0 kg. Jay's motion can be considered simple harmonic motion.



(a) Calculate the maximum velocity of Jay and the swing.

(b) Use a reference circle or other method to determine how much time Jay's displacement is greater than 0.200 m from equilibrium over one period.

(c) Jay stops swinging her legs when the swing is at its maximum displacement. On the grid sketch a graph of her displacement over the next three periods. Include values for the time and for the initial displacement at t = zero.



QUESTION THREE (2018;3)

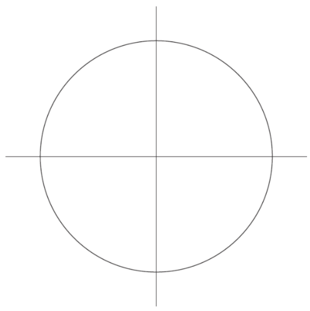
When astronauts return to Earth, a spring under their seat reduces the force during the landing. The astronaut's kinetic energy is converted to spring potential energy as the spring is compressed. If friction is negligible, this will set the astronaut into simple harmonic motion.

(a) State the conditions required for the astronaut's motion to be considered simple harmonic motion.

During a landing, an astronaut and seat had a combined mass of 80.0 kg and were set into a simple harmonic motion with an amplitude of 0.150 m and a period of 0.940 s.

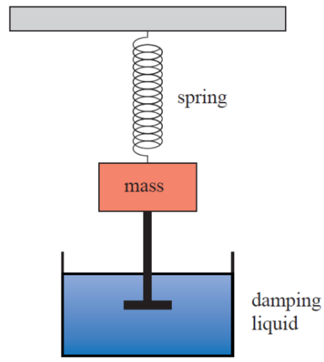
(b) Determine the spring constant of the spring and the amount of energy stored in the spring at maximum displacement.

(c) Using a reference circle or otherwise, determine the velocity of the astronaut when the astronaut is 0.100 m above the equilibrium position (Assume the motion is un-damped).



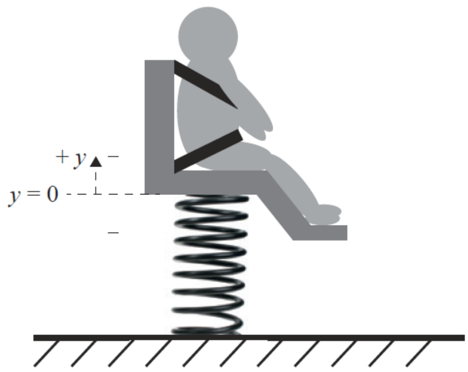
(d) The motion of the astronaut is quickly brought to rest by a damping system on the spring. Damped harmonic motion can be modelled in the laboratory with a mass, spring, and beaker of water as shown. Discuss how damping will affect the amplitude and period of the harmonic motion of the mass on the spring. Your discussion should include:

- a description of what is meant by damping
- a description of the damping force in the laboratory model and how this will impact the motion
- a sketch of a graph of the position of the mass versus time starting at the moment the mass is released from the maximum downward displacement.



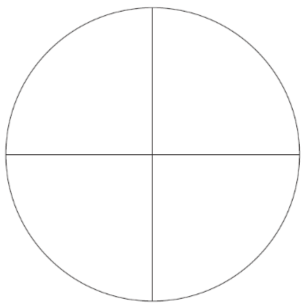
QUESTION THREE (2017;3)

Astronauts need to be able to measure their mass regularly so that they can monitor their health. They can do this by being strapped on to a lightweight seat that is attached to a spring as shown in the diagram below. When Sylvia is displaced from equilibrium, she oscillates in simple harmonic motion with a period of 8.00 s. You may assume her motion is linear.

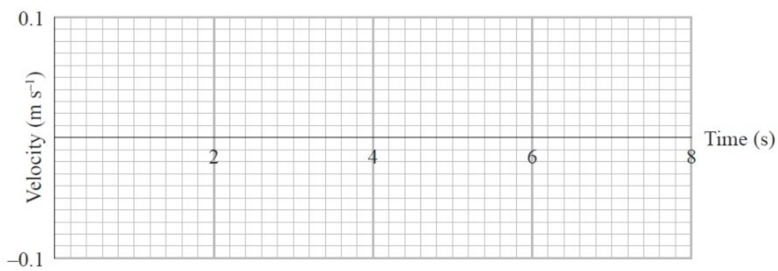


(a) Describe any changes in the size or direction of the restoring force as Sylvia moves away from equilibrium in the +y direction.

(b) The amplitude of Sylvia's oscillation is 0.120 m. Use a reference circle or other method to calculate the shortest time it takes for Sylvia to move up 0.080 m from her equilibrium position.



(c) On the axis below, draw a graph showing Sylvia's velocity vs time, starting when she is closest to the floor. Include the value of the maximum velocity.



(d) To start the oscillation, Sam applies a vertical force of 4.40 N to Sylvia. This force causes Sylvia to move a distance of 0.120 m.

- Calculate Sylvia's mass.
- Describe any assumptions you have made to simplify your calculation.

SIMPLE HARMONIC MOTION (2016;3)

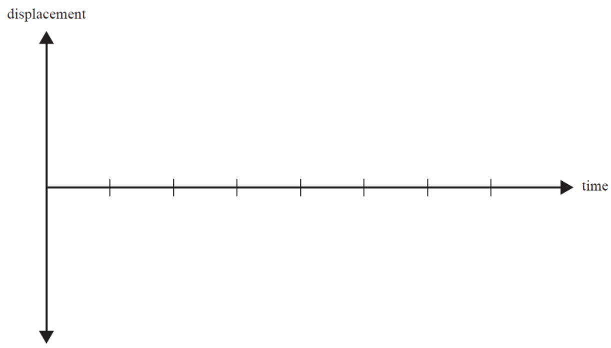
A toy bumble bee hangs on a spring suspended from the ceiling in the laboratory. Tom pulls the bumble bee down 10.0 cm below equilibrium and releases it. The bumble bee moves in simple harmonic motion.

(a) State the two conditions necessary for simple harmonic motion.

(b) The bumble bee's oscillation has a period of 1.57 s. Calculate the bumble bee's acceleration at time t = 0.25 s after Tom releases the bumble bee from the lowest point.

(c) Tom pushes the toy bumble bee with a very small force at regular intervals of time (periodically), so that eventually it is moving up and down with a very large amplitude. State the name of this phenomenon. Explain how the bumble bee's motion develops a very large amplitude.

(d) Tom stops pushing the bumble bee when its displacement is 20 cm. Using the axes given below, draw a graph of displacement against time for three complete oscillations, starting from y = +20 cm. Include appropriate values on both axes.



GRAVITY ELEVATORS (2015;2)

Earth's average radius = 6.38×10^6 m.

In the 2012 science fiction movie *Total Recall,* a gravity- powered elevator called "The Fall" is used to transport passengers between the Northern and Southern hemispheres, straight through the Earth. If a straight tunnel could be dug through the Earth from the North Pole to the South Pole, protected from the heat inside the Earth and the journey unaffected by friction, an elevator could be used, harnessing the gravity of the planet.

Once dropped, the elevator would accelerate downwards and then decelerate once it had passed through the midpoint and -in the absence of friction - would just arrive at the far side of the Earth.

An equation can be used to summarise acceleration of the elevator.

$a = -1.54 \times 10^{-6}$ y, where y = distance from the midpoint

(a) One of the passengers on the elevator stands on bathroom scales at the start of the journey. Describe why the bathroom scales read zero.

(b) Calculate:

(i) The maximum acceleration of the elevator.

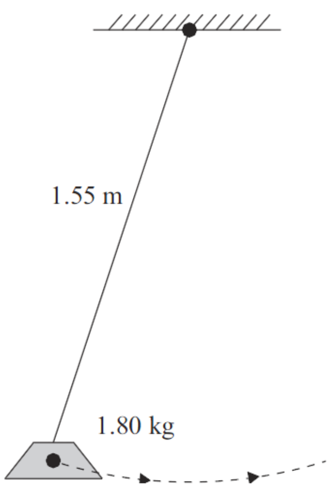
(ii) The maximum linear velocity of the elevator.

(c) Explain how the equation given shows that the elevator is undergoing simple harmonic motion.

(d) Calculate the time the journey from the North Pole to the South Pole would take.

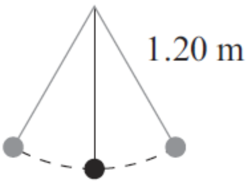
THE PENDULUM (2014;2)

A pendulum is set up, as shown in the diagram. The length of the cord attached to the bob is 1.55 m. The bob has a mass of 1.80 kg. The acceleration due to gravity of Earth = 9.81 m s^{-2}.



(a) Calculate the time it takes for the pendulum bob to swing from one side to the other.

OSCILLATING BALLS (2013;3)



A ball, attached to a cord of length 1.20 m, is set in motion so that it is swinging backwards and forwards like a pendulum.

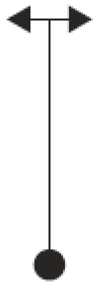
(a) Show that the period of a pendulum of length 1.20 m that is oscillating in simple harmonic motion is 2.20 s.

(b) Explain what must be done to ensure that the motion of the ball approximates simple harmonic motion.

(c) On the axes, sketch a graph to show what happens to the ball's total energy over time until it stops swinging.

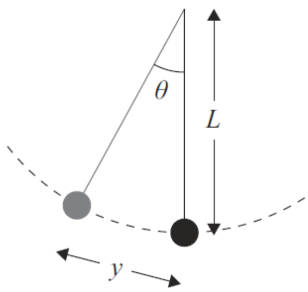
(d) It is possible to get the ball swinging by holding the top end of the cord and gently shaking it backwards and forwards. Explain how shaking the top end of the cord can make the ball on the bottom of the cord oscillate in simple harmonic motion.

In your answer, you should consider resonance and energy transfer.

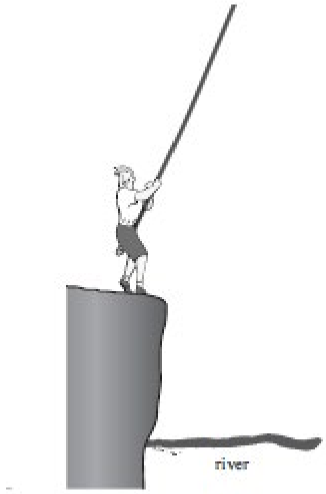


(e) Simple harmonic motion requires a restoring force that changes in proportion to the size of the displacement. Discuss what provides the restoring force when the ball is swinging in simple harmonic motion. In your answer, you should:

- describe what forces act on the ball
- explain how these forces change as the ball swings
- draw vectors to show how a restoring force is produced



SIMPLE HARMONIC MOTION (2012;2)



Daniel is having fun on a swing that consists of a rope with one end tied to the branch of a tree, and a wooden bar attached to the other end. The rope hangs down over the edge of a bank of a river.

If Daniel lifts his feet without pushing, he swings out, over the river, in simple harmonic motion.

The angular frequency of his motion is 1.45 rad s^{-1}, and he travels a distance of 0.80 m before reaching the equilibrium position.

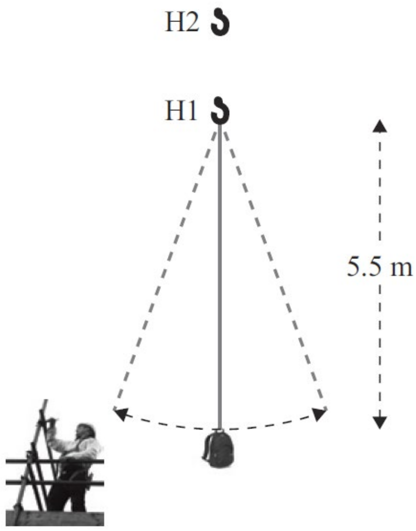
(a) Calculate the acceleration of Daniel's simple harmonic motion at the instant he lifts his feet.

(b) Calculate the distance Daniel travels in 1.8 s after he lifts his feet.

(c) On another occasion, instead of lifting his feet, Daniel pushes off from the starting point. Discuss what effect this could have on the motion of his swing. Your answer should include an explanation for why the amplitude of his swing increases.

(d) Sometimes Daniel sits on the wooden bar and sometimes he stands. Discuss how the period of the motion would be affected by the way Daniel rides on the swing.

SIMPLE HARMONIC MOTION (2011;2)



Brad is working on a scaffold. He ties his bag to a hook, H1, using a light rope of length 5.50 m, as shown in the diagram. He pulls the bag 0.900 m to the side and lets it swing.

(a) Show that the time period of the bag's swing is 4.70 s.

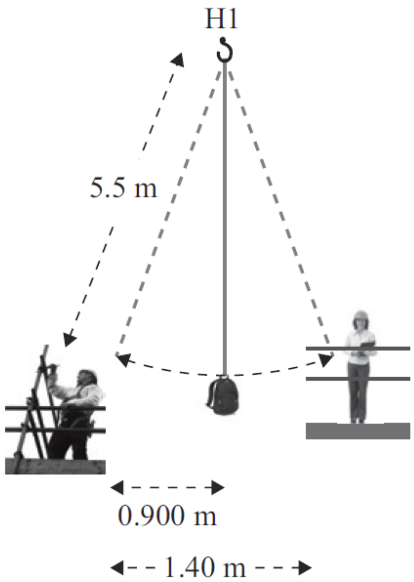
(b) (i) Calculate the maximum velocity of the swinging bag.

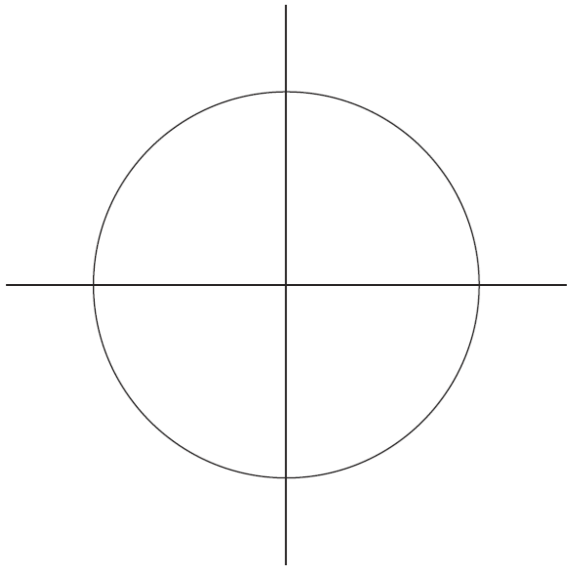
(ii) Show (on the diagram above) where this maximum velocity occurs.

(c) Explain how the tension in the rope varies as the bag swings.

(d) Explain how the maximum velocity of the bag would change if Brad had used a longer rope to attach his bag to the hook H2. *Assume that the bag is given the same initial displacement.*

(e) Brad offers his friend Ann a sandwich from his bag. He lets the bag swing when Ann is 1.4 m away. He calls to her when the bag has travelled 0.100 m. Ann's reaction time is 0.400 s. Calculate the time between Brad's call and when the bag reaches Ann. *Use the reference circle below, or some other method.*





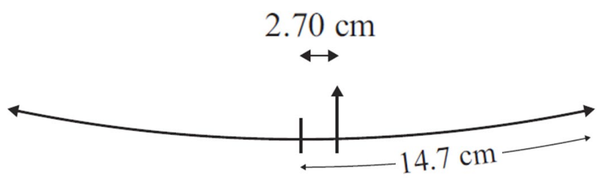
SIMPLE HARMONIC MOTION (2010;3)

One way to measure the rotational inertia of a bicycle wheel (about its axis) is to set it up as a pendulum by hanging it from a string, as shown in the diagram. The oscillations of the wheel can be approximated to simple harmonic motion.

(a) What condition must apply to the size of the acceleration of any object that, at any instant, is moving with simple harmonic motion?

(b) The spokes cause friction with the air. Describe and explain how this will affect how the amplitude of the oscillations changes with time.

A wheel is hung as shown in the diagram opposite. When the pendulum is set in motion, its oscillations have amplitude 14.7 cm and angular frequency 2.40 rad s^{-1}.



(c) The angular frequency value is calculated from the period of the simple harmonic motion. An accurate value for the period is found by timing a large number of oscillations. Calculate the number of oscillations that are measured in a total time of 83.8 s.

(d) In order to time the oscillations, the stopwatch is operated as the centre of the wheel passes a marker. This marker is 2.70 cm from the equilibrium position.

(i) The wheel has a mass of 1.65 kg. Calculate the restoring force acting on the wheel as it passes the marker.

(ii) Calculate the speed of the wheel at the instant it passes the marker.

QUESTION THREE (2009;3)

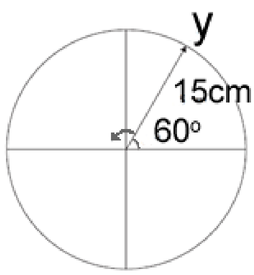
Acceleration due to gravity = 9.81 ms^{-2}

The suspension in Sam's plane contains large springs that make landing on rough airstrips more comfortable. The uncompressed length of the springs is 55.0 cm. When they are placed on the plane (of mass 635 kg), they are compressed to a length of 37.5 cm.

(a) Show that the force constant (k) of the springs is 3.56×10^4 N m^{-1}

(b) When the plane goes over a bump, it starts oscillating with vertical simple harmonic motion. Show that the period of the simple harmonic motion is 0.839 s.

The phasor diagram below shows the displacement phasor at one instant, at an angle of 60° when the amplitude of the simple harmonic motion is 15 cm.



(c) By drawing the velocity phasor on the diagram or by another method, describe the vertical velocity of the plane at this instant.

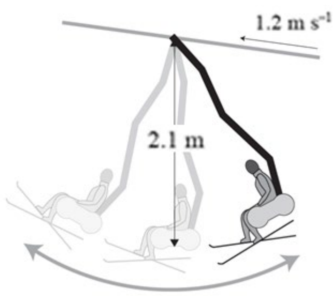
(d) Calculate the vertical acceleration of the plane at the instant shown in the phasor diagram.

(e) Shock absorbers help improve the comfort for passengers by damping the simple harmonic motion.

Describe the effect of moderate damping on the displacement and acceleration of the oscillating plane, and discuss how damping will affect the discomfort felt by a passenger. You may use the grid provided to help illustrate your answer.

SIMPLE HARMONIC MOTION (2008;3)

Acceleration due to gravity = 9.81 m s^{-2}



Chairlifts carry skiers to the top of the mountain by way of a continuously-moving steel cable to which the chairs are attached. As soon as the skiers sit down, the chair is lifted from the ground by the cable. The chair swings back and forth for a while. The diagram below shows a side view of a chair as it oscillates (amplitude not to scale).

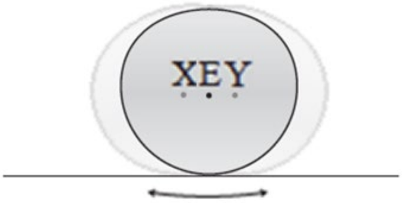
The swinging motion of the chair and skier approximates to a simple pendulum 2.1 m long with a mass of 2.0×10^2 kg.

(a) Show that the period of swing of the pendulum is 2.9 s.

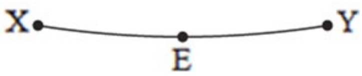
QUESTION THREE (2007;3)

Acceleration due to gravity = 9.81 m s^{-2}.

If the bowl is placed on a hard, flat surface, it will rock back and forth for a while, as shown in the diagram below.



While the bowl is rocking, the motion of the centre of mass of the bowl is modelled as simple harmonic motion.



(a) If a bowl completes exactly 13 cycles of its rocking motion in 9.32 s, calculate the period of the simple harmonic motion. Give your answer to the correct number of significant figures.

It can be shown that an approximate expression for the period of the simple harmonic motion is

$T = 2\pi\sqrt{\frac{R}{g}}$ where R is the radius of curvature of the path (XEY in the diagram) of the centre of mass, and g is the acceleration due to gravity.

(b) Calculate the radius of curvature of the path of the centre of mass.

The rocking motion of the bowl is due to the action of a restoring force on its centre of mass.

(c) State one of the conditions that must apply to this restoring force if the rocking motion is simple harmonic motion.

(d) The approximate size of this restoring force is given by

$$F_R = \frac{F_g y}{R}$$

where R is the radius of curvature of the path of the centre of mass, and y is the displacement of the centre of mass from its equilibrium position, E.

By using the general equation for simple harmonic motion, show how this expression $T = 2\pi\sqrt{\frac{R}{g}}$ can be derived. The bowl does not keep rocking; its motion is acted on by opposing force(s).

(e) Describe the force(s) that could cause the motion of the bowl to be damped.

(f) Explain why the bowl will come to rest more quickly on a soft surface. Your explanation should include how it loses its energy.

LUCY ON HER SWING (2006;1)

Strength of gravity = 9.81 N kg^{-1}

Little Lucy loves playing on her homemade swing. It is a seat attached to a tree by a rope, and her mum pushes her to make her swing. The swing acts like a simple pendulum and Lucy's mum pushes her gently so that her swinging motion can always be considered to be simple harmonic motion. Lucy has a mass of 31 kg. The angular frequency of Lucy's simple harmonic motion is 2.2 rad s^{-1}.

(a) Using the information given above, calculate the frequency of Lucy's motion.

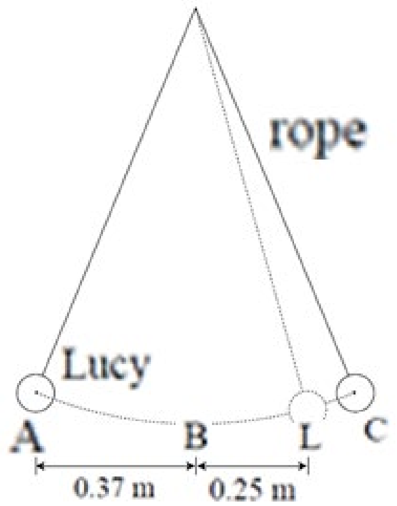
(b) The period of Lucy's simple harmonic motion can be calculated to be 2.9 s. Calculate the length of the rope used to make the swing.

(c) Lucy's mum knows that, as Lucy grows, the length of the rope will have to be shortened. If the length of the rope is halved, explain what effect this would have on the period of Lucy's swings.

Lucy's mum stops pushing her, and leaves her to swing freely by herself. Assume the rope is at its original length and that Lucy swings with constant amplitude of 0.37 m.

(d) Calculate Lucy's maximum acceleration.

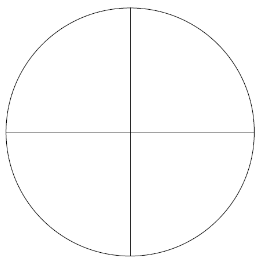
The diagram below shows Lucy swinging. At position L, she is travelling towards C, and is 0.12 m from C.



(e) On the diagram above, draw an arrow to show the direction of the net force acting on Lucy when she is at position L.

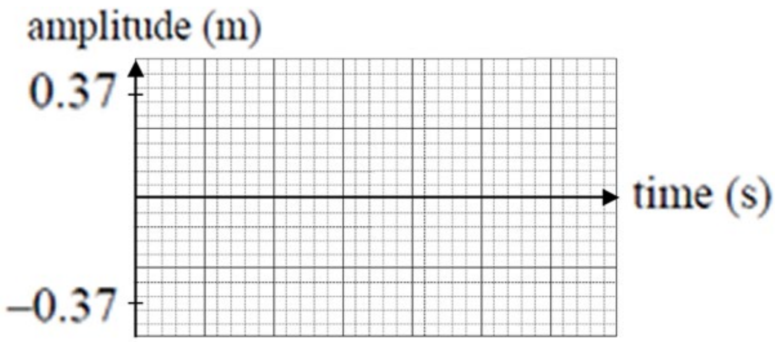
(f) Calculate the size of the net force acting on Lucy at position L.

(g) Using the reference circle, or any other method, calculate the time it would take Lucy to travel from L to C and back to A. Assume the amplitude of her swing remains constant at 0.37 m.



In practice there will be a significant energy loss due to friction as Lucy continues to swing.

(h) On the axes, sketch a graph of her amplitude against time for three oscillations. Values are not required to be marked on either axis.

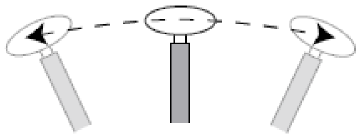


SIMPLE HARMONIC MOTION (2005;2)

The London Eye is a giant rotating wheel that has 32 capsules attached at evenly spaced intervals to its outer rim. Passengers riding in the capsules get spectacular views over London, especially at the top.

Capsule

Wind can make the top of the wheel sway from side to side. The maximum total distance a capsule moves from one side to the other is restricted to 0.150 m.

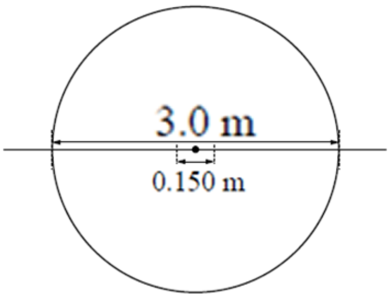


The diagram shows just one capsule (at the top of the wheel) as it sways from side to side. Assume the swaying motion is simple harmonic with a natural angular frequency of 1.8 rad s^{-1}.

(a) What is the amplitude of the motion?
(b) Calculate the maximum acceleration of a passenger. Give your answer to the correct number of significant figures.
(c) At what displacement from the equilibrium position would this maximum acceleration occur?

The restriction on the maximum amplitude of vibration is brought about by dampers that are incorporated into the rim structure. Without the dampers, strong wind could cause the top of the wheel to sway a total distance of as much as 3.0 m from one side to the other. This would be uncomfortable for the passengers. (Assume the natural frequency of the undamped SHM is the same as the damped SHM.)

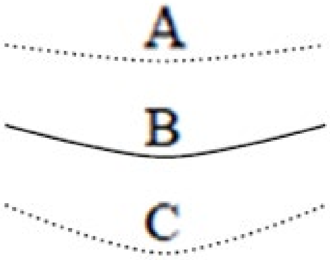
(d) If the dampers were not present, calculate the speed of the capsule 0.75 s after it had reached an end position.
(e) In questions (b) and (c), the acceleration was calculated at a particular displacement with the dampers present. Explain why the acceleration would have had the same value at this displacement if the dampers had not been present.
(f) Calculate the period of the SHM.
(g) If the dampers were not present the passengers would sway beyond the maximum damped displacement. Using the reference circle (not to scale) or otherwise, calculate the time a passenger would spend **outside** the damped maximum displacement each cycle.



BOUNCING ON THE TRAMPOLINE (2004;2)

Use the gravitational field strength = 9.80 N kg^{-1} (or acceleration due to gravity = 9.80 ms^{-2}).

Ivan is practising for a national trampolining competition. After a while he takes a rest and just stands on the mat, allowing it to bounce him up and down. Assume this motion is like a mass on a spring, performing simple harmonic motion.

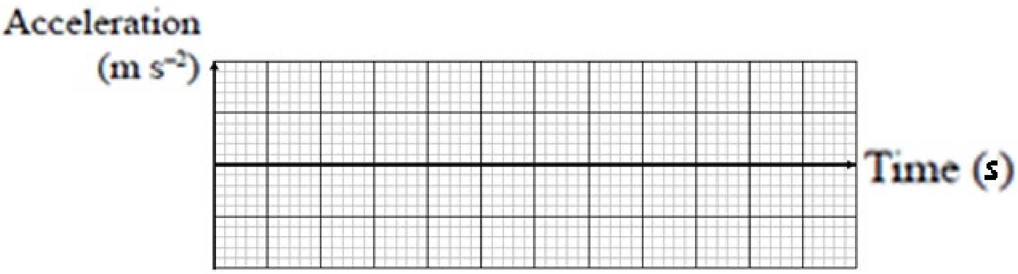


In each oscillation, the surface of the trampoline mat moves between positions A and C as shown in the diagram. Initially the amplitude (maximum displacement) of the oscillation is 0.14 m.

(a) Calculate the distance AC.
(b) Ivan's mass is 62.0 kg and the period of his oscillations is 0.85 s. Show that the spring constant of the trampoline has a value of 3387.8 Nm^{-1}. Give this answer to the appropriate number of significant figures.
(c) Calculate the angular frequency for the motion.

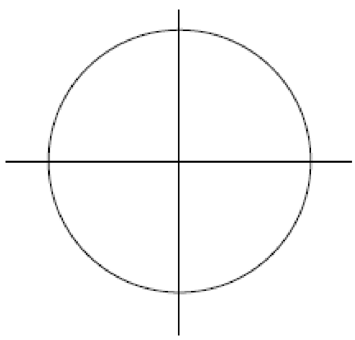
Positions B, A and C show the middle and end positions of the oscillations of the trampoline mat.

On the axes provided, sketch the shape of the acceleration-time graph of Ivan's motion for one complete cycle. The maximum acceleration is 7.6 ms^{-2}. Label at least one point on the time axis.



Assume t = 0 when Ivan is at point C (Ignore any loss of energy in the cycle.).

(d) (i) State a position at which Ivan's kinetic energy is maximum.
(ii) Calculate the speed that gives this maximum kinetic energy.
(e) Calculate Ivan's potential energy at position C in his motion.
(f) (i) On the reference circle, show Ivan's position after one-eighth of a cycle. Label the distance of this position from the equilibrium position, y. Again assume t = 0 at point C.
(ii) Calculate Ivan's distance, y, from the equilibrium position at this time.



(g) By working his legs, Ivan can input energy and increase the maximum distance from the equilibrium position of his bounce until his feet leave contact with the mat for part of each cycle. Explain why his motion can no longer be considered as simple harmonic.

(h) On one occasion, Ivan's speed as he leaves the mat is 2.4 ms^{-1} His potential energy at this instant is 54.5 J. Calculate the maximum distance from the equilibrium position that Ivan must generate to give him this speed as he leaves the mat. (Assume Ivan does not input energy on the way up.)

(i) Ivan needs to bounce at a particular frequency to get the greatest amplitude of oscillation on the mat. What is the name of this frequency?