Level 3 Physics: Demonstrate understanding of mechanical systems – Simple Harmonic Motion - Answers

In 2013, **AS 91524** replaced **AS 90521.**

The Mess that is NCEA Assessment Schedules….

In AS 90521 there was an Evidence column with the correct answer and Achieved, Merit and Excellence columns explaining the required level of performance to get that grade. Each part of the question (row in the Assessment Schedule) contributed a single grade in either Criteria 1 (Explain stuff) or Criteria 2 (Solve stuff). From 2003 to 2008, the NCEA shaded columns that were not relevant to that question.

In 91524, from **2013 onwards**, each part of a question contributes to the overall Grade Score Marking of the question and there are no longer separate criteria. There is no shading anymore. There is no spoon. At least their equation editor has stopped displaying random characters over the units.

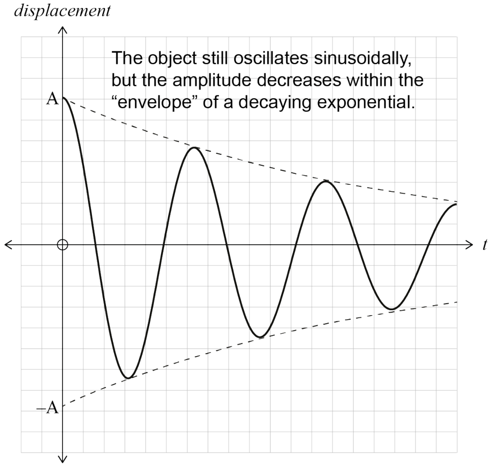
For 2019:

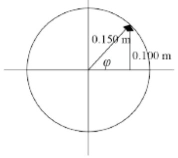
NØ	N1	N2	A3	A4	M5	M6	E7	E8
0	1A	2A or 1M	3A or 1A +1M or 1E-	4 A or 2A + M or 2M or 1A+1E-	1A + 2M or 1M+1E- or 3A +1M or 2A + 1E-	2A + 2M or 3M or 3A + 1E- or 1A +1M + 1E-	2M+1E- or 2A +1M + 1E- or A + 2M + 1E-	A + 2M +E

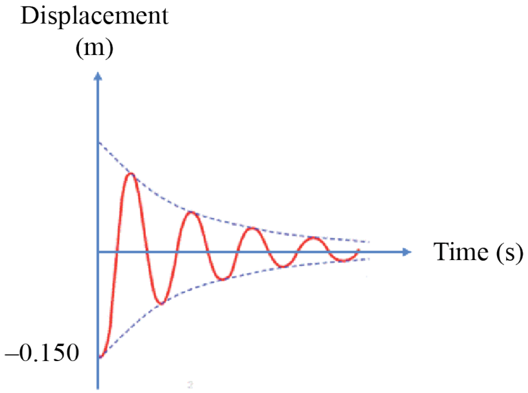
Other combinations are also possible using a=1, m=2 and e=3. However, for M5 and M6, at least one Merit question needs to be correct (maximum 6). For E7 or E8, at least one Excellence needs to be correct (maximum 8). **Note: E- and E only applies to the E7 and E8 decision. <----** It's good that NCEA gets clearer and clearer after 16 years!

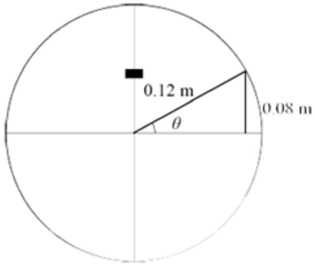
Question	Evidence	Achievement	Merit	Excellence
2019(3) (a)	$v_{max} = Aw = 0.310\text{ m} \times \frac{2\pi}{2.40} = 0.811578\text{ m s}^{-1} = 0.812\text{ n}$	• Correct answer.		

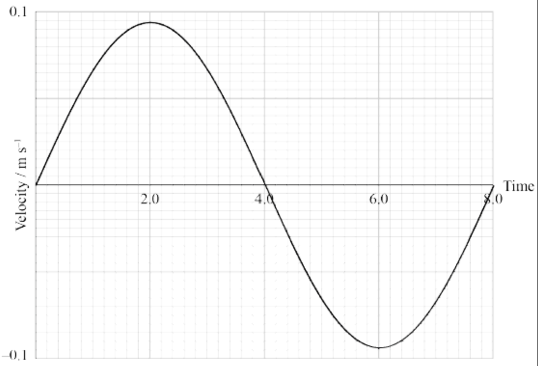
(b)	Time to get from equilibrium to 0.200 m: $y = A\sin wt$ 0.200 m = 0.310 m × sin(2.618t) t = 0.2679 s Time displacement is LESS than 0.200 m = 4t = 4 × 0.2679 s = 1.071 s Time displacement is GREATER than 0.20[cut off] = 2.40 s – 1.071 s = 1.33s OR y = 0.200m and A=0.310m drawn on ref cir[cut off] $\theta = \cos^{-1}\left(\frac{0200}{0.310}\right) = 49.22°$ $t = \left(\frac{49.822°}{90°}\right) \times 2.4\text{ s} = 0.33\text{ s}$ OR $y = \cos\left(\frac{0200}{0.310}\right)$ $\theta = \cos^{-1}\left(\frac{0200}{0.310}\right) = 0.86956\text{ rad}$ $\theta = \omega t$ $t = \frac{\theta}{\omega} = \frac{0.86596}{2.618} = 0.33\text{ s}$ Time displacement greater than 0.200 m $= 0.33\text{ s} \times 4 = 1.33\text{ s}$	• Correct setup of reference circle for amplitude and displacement. OR Correct angular frequency ω = 2.618$rads$!"used in the calculation.	• Correct answer and working.	

(c)	Starting at max. displacement = +/– 0.310 m with values on graph for 3 periods (2.4/4.8/7.2), three periods only, displacement showing exponential decay. 	• Correct starting max and shape showing decay. OR • Correct time values and shape showing decay *NOTE: Must have:* *(1) Three full cycles* *(2) Exponential decay*	Correct damped shape for 3 complete cycles, constant period, correct starting max and at least one value on each of the axes (A = 0.310 m, and T = 2.4 s)	
2018(3) (a)	Restoring force always acts toward the equilibrium p to the displacement from the equilibrium position. OR Acceleration always acts toward the equilibrium and is directly proportional to the displacement from the equilibrium position. OR Accept equations in the explanation e.g. ($a = -w^2y$, a $\alpha - y$. $F\ \alpha - y$, F = $-ky$ etc.)	Correct answer.		

(b)	$T = 2\pi\sqrt{\frac{m}{k}}$ $0.940\text{ s} = 2\pi\sqrt{\frac{80.0\text{ kg}}{k}}$ $k = 3574\text{ N m}^{-1} = 3570\text{ N m}^{-1}$ $E = \frac{1}{2}kx^2 = 0.5 \times 3574\text{ N m}^{-1} \times (0.150\text{ m})^2 = 40.2\text{ J}$	k = 3570 N m^{-1} OR E_{spring} = 40.2 J OR Correctly calculates Elastic potential energy with incorrect spring constant (k).	All correct.	
(c)	Determines t using the reference circle. $\phi = \sin^{-1}\left(\frac{0.100}{0.150}\right) = 0.7297$ radians  (Accept diagram with phasor on left side of circle at same height.) $\omega = \frac{2\pi}{T} = 6.684\text{ rad s}^{-1}$ $\omega = \frac{\theta}{t}$ $t = \frac{0.7297\text{ radians}}{6.684\text{ rad s}^{-1}}$ $= 0.109\text{ s}$ $v = A\omega\cos(\omega t)$ $= 0.15 \times 6.684 \times \cos(6.684 \times 0.109)$ $= 0.748\text{ m s}^{-1}$ OR $v_{max} = \omega A$ $= \frac{2\pi}{T}A$ $= \frac{2\pi \times 0.15}{0.94}$ $= 1.003\text{ m s}^{-1}$ $\theta = \sin^{-1}\left(\frac{0.1}{0.15}\right) = 41.81°$ or 0.7297 rad Degrees $v = v_{max}\cos\theta$ $= 1.003 \times \cos 41.81° = 0.748\text{ m s}^{-1}$ Radians $v = 1.003 \times \cos 0.7297 = 0.748\text{ m s}^{-1}$	Correct setup of reference circle for amplitude and displacement. OR Correct reference circle without arrowheads on the phasors. OR Correct selection of equation **and** substitution. OR Found the correct maximum velocity. OR Calculated the correct angle.	Correct answer supported by working.	

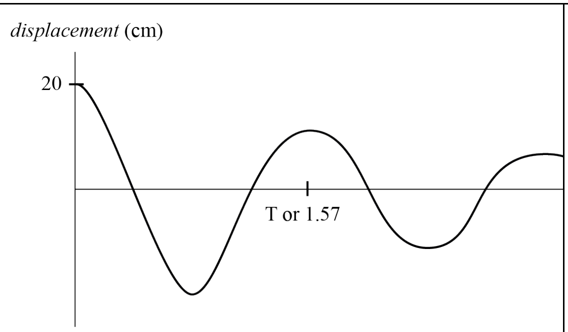
(d)	Damping means a force acts in the opposing direction to the restoring force, removing energy from the system and thus reducing the amplitude of a harmonic motion. In the laboratory model, on the way upward after being released, the damping liquid provides a downward frictional force working against the upward restoring force of the spring. Once the mass reaches the top and begins to move downward, the water provides an upward, frictional force opposing the downward restoring force of gravity and the spring. The period and frequency of the motion will not be changed as these depend on the mass and the spring constant only. The damping force will transfer energy away from the system, so the amplitude will be reduced with each cycle.  	Reasonable description of damping. OR Mentions reduced amplitude. OR Mentions constant period (and / or frequency). OR Identifies damping force as friction with the liquid. OR Reasonable attempt at a graph showing decreasing amplitude.	Discusses THREE of the achieved aspects.	Full description. (E) NOTE: If the graph does not start from the minimum point but has a steady period with decreasing amplitude then the maximum mark is E–. Note: Allow **E–** if there is one minor feature missing from the written answer, but it is shown on the graph. For example, the period is constant (shown clearly on the graph) and the amplitude decreases.

2017(3) (a)	The direction of the restoring force does not change OR the direction of the restoring force remains down OR the restoring force increases as she moves away from equilibrium OR Accept equations in the explanation (F = -ky) or (F α –y) to this particular situation.	Correct answer.		
(b)	In degrees: $\sin\theta = \frac{0.08}{0.12}$ $\theta = 41.8°$ $\frac{t}{T} = \frac{\theta}{360°}$ $t = \frac{41.81° \times 8.00}{360°}$ $t = 0.928 = 0.93\text{ s}$  In Radians: $\sin\theta = \frac{0.08}{0.12}$ $\theta = 0.73\text{ rad}$ $t = \frac{\theta}{\omega}$ and $\omega = \frac{2\pi}{T}$ $t = \frac{\theta T}{2\pi} = \frac{0.73 \times 8.00}{2\pi}$ $t = 0.929 = 0.93\text{ s}$ OR $y = A\sin\omega t$ and $\omega = \frac{2\pi}{T}$ Note: $\omega = 0.785\text{ rads}^{-1}$ $0.08 = 0.12\sin\left(\frac{2\pi t}{8.00}\right)$ $\frac{2\pi t}{8.00} = \sin^{-1}\left(\frac{0.08}{0.12}\right)$ $\frac{2\pi t}{8.00} = 0.7297\text{ rad}$ $t = \frac{0.7297 \times 8.00}{2\pi}$ $t = 0.929 = 0.93\text{ s}$	Correct diagram. OR Correct answer with insufficient working. OR Correct angle in either degrees or radians OR Used appropriate equation e.g. $y = A\sin\omega t$ OR Correct angular frequency $\omega = 0.785\text{ rad s}^{-1}$	Correct calculation and answer	

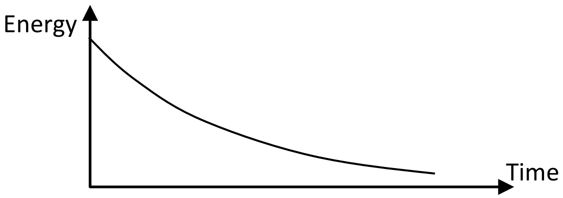
(c)	$v_{max} = \omega A$ $v_{max} = \frac{2\pi}{T} A$ $v_{max} = \frac{2 \times \pi \times 0.12}{8} = 0.0942777$ $v_{max} = 0.094 \text{ m s}^{-1}$ 	Correct answer for v_{max}. OR Graph correct shape.	Correct answer for v_{max}. AND Graph correct shape.	

(d)	$F = kx$ $k = \frac{F}{x} = \frac{4.4}{0.12} = 36.66 \text{ N m}^{-1}$ $T = 2\pi\sqrt{\frac{m}{k}}$ $8.00^2 = 4\pi^2 \frac{m}{36.66}$ $m = 59.4 \text{ kg}$ OR $E_p = \frac{1}{2}kx^2 = \frac{Fx^2}{2x} = \frac{Fx}{2} = \frac{4.40 \times 0.120}{2} = 0.264 \text{ J}$ $E_p = E_k = \frac{1}{2}mv^2$ $= \frac{1}{2}m(0.0942777)2 = 0.264$ $m = 59.4 \text{ kg}$ *OR* $a_{max} = \omega^2 A$ $a_{max} = \left(\frac{2\pi}{8}\right)^2 \times 0.12$ $a_{max} = 0.074 \text{ rad s}^{-2}$ $m = \frac{F}{a_{max}} = \frac{4.4}{0.074}$ $m = 59.4 \text{ kg}$ The period of oscillation depends on the total mass $T = 2\pi\sqrt{\frac{m}{k}}$. This formula states that if the mass is increased, the period increases also. In order to get the true mass of the astronaut, the mass of the seat and the effective mass of the spring should be omitted. However, because this information is not known and because the question states that the seat is "lightweight", the mass of the seat (and spring) can be omitted for simplicity. OR For using the energy conversion to find mass, candidate justifies WHY it is appropriate to equate kinetic energy to elastic potential energy, i.e. negligible heat losses so energy is conserved.	Correct spring constant. OR Selected the correct period equation. OR Correct energy calculation OR Calculates the maximum acceleration as 0.074 rad s^{-2}. OR States that mass of the seat and / or spring is ignored. OR States that energy is conserved if using energy conversion to find m	Allow M for follow on error for k. OR Allow M for follow on error for v using conservation of energy method. OR Allow M for the follow-on error for a_{max} using Newton's second law. OR Simplifying assumption described. E.g. mass of seat (and spring) is ignored because the question stated that the seat was "lightweight". OR Candidate describes assumption that kinetic is the same size as the elastic potential energy.	Correct calculation and correct answer for the total mass AND Two supporting assumptions described (E) Correct calculation and correct answer for the total mass. (E-) OR Three or more simplifying assumptions described and linked. (E-) e.g. Simplifying assumption described. E.g. mass of seat (and spring) is ignored because the question stated that the seat was "lightweight" and because this self-mass is ignored, we get a period lower than expected.

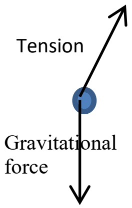
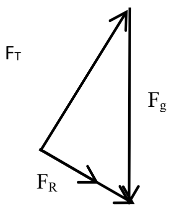
2016(3) (a)	The acceleration (or restoring force) is proportional to displacement and is (acts) in the opposite direction to displacement. Accept equations in the explanation e.g. ($a = -\omega^2 y$), ($a \propto -y$), ($F = -ky$) or ($F \propto -y$)	Both points stated.		
(b)	When $T = 1.57$ s, $\theta = 360°$ When $t = 0.25$ s, $\theta = 57.3°$ $\omega = \frac{2\pi}{T} = 4.00 \text{ rad s}^{-1}$ $a = -A\omega^2 \cos\theta$ $= -0.10 \times 4.00^2 \cos 57.3 = 0.864 \text{ m s}^{-2}$ OR $\omega = \frac{2\pi}{T} = 4.00 \text{ rad s}^{-1}$ $= -A\omega^2 \cos\omega t$ $= -0.10 \times 4^2 \cos(4 \times 0.25)$ $= -0.864 \text{ m s}^{-2}$	Correct angle for $t = 0.25$ s. OR Correct value for angular frequency. OR Use of suitable equation $a = -A\omega^2 \cos\theta$	One error in calculation. e.g.: Forgot to square ω. OR Used 10.0cm instead of 0.10m.	Correct calculation and correct answer for acceleration with unit. (E) Correct calculation and answer for acceleration without unit. (E-)
(c)	This is an example of resonance. Tom must put energy into the system (the driving frequency) at the natural frequency. The natural frequency is the frequency at which the bumble bee will normally oscillate. (Amplitude increases until energy input equals kinetic energy lost as heat.)	Idea of resonance or that the frequency of the driver matches the natural frequency of the spring. OR "Resonance" stated but not explained.	Idea of resonance including the word "Resonance". AND Links matching driving frequency to natural frequency. OR Explains maximum amplitude.	

(d)		Correct damped shape starting from +y for 3 complete cycles. OR Constant amplitude if no damping statement given for 3 complete cycles.	Correct damped shape for 3 complete cycles and constant period and at least one value on axes (A_0 = 20 cm, and T = 1.57 s). OR States assumptions of zero damping and the graph shows this for 3 complete cycles. Constant period and at least one value on axes (A_0 = 20 cm, and T = 1.57 s).	
2015(2) (a)	The passenger and the bathroom scales are falling at the same rate, so there are no contact forces between them. *Accept support/normal forces/reaction force and free falling.*	Correct description.		
(b)	$a = -1.54 \times 10^{-6}$ y, $y = A$ and $\omega^2 = 1.54 \times 10^{-6}$ $a_{max} = -A\omega^2 = 1.54 \times 10^{-6} \times 6.38 \times 10^6$ = 9.83 m s^{-2} OR Maximum acceleration at Earth's surface is g = 9.81 m s^{-2}.	Use of suitable equation e.g. $v = -A\omega^2 \sin \omega t$ OR Maximum acceleration at Earth's surface is g.	$a = -A\omega^2$ $= 1.54 \times 10^{-6} \times 6.38 \times 10^6$ = 9.83 m s^{-2} OR Maximum acceleration at Earth's surface is g = 9.81 m s^{-2}	
(c)	$\omega = \sqrt{1.54 \times 10^{-6}}$ rads^{-1} $V_{max} = A \times \omega = 6.38 \times 10^6 \times 1.24 \times 10^{-3}$ = 7910 m s^{-1}. OR $\omega^2 = \frac{g}{R_{Earth}}$, $\omega = \sqrt{\frac{g}{R_{Earth}}}$, $A = R_{Earth}$ $V_{max} = A\omega = \sqrt{g.R_{Earth}}$ $V_{max} = \sqrt{9.81 \times 6.38 \times 10^6} = 7910 \text{ m s}^{-1}$	Use of suitable equation e.g. $v = -A\omega\cos\omega t$ OR $V_{max} = A\omega = \sqrt{g.R_{Earth}}$	$v = A \times \omega$ $= 6.38 \times 10^6 \times 1.24 \times 10^{-3}$ = 7910 m s^{-1} OR $V_{max} = A\omega = \sqrt{g.R_{Earth}}$ $V_{max} = \sqrt{9.81 \times 6.38 \times 10^6} = 7910 \text{ m s}^{-1}$	

(d)	$f = \frac{1}{T}$ and $\omega = 2\pi f$ and $\omega^2 = 1.54 \times 10^{-6}$ so $T = \frac{2\pi}{\omega}$ $T = \frac{2\pi}{\sqrt{1.54 \times 10^{-6}}}$ $T = 5063$ s $= 84$ minutes And journey will take half of one time period so the time = 42 minutes. OR $\omega^2 = \frac{g}{R_{Earth}}$, $\omega = \sqrt{\frac{g}{R_{Earth}}}$, $A = R_{Earth}$ $T = \frac{2\pi}{\omega}$ $T = 2\pi\sqrt{\frac{R_{Earth}}{g}} = 2\pi\sqrt{\frac{6.38 \times 10^6}{9.81}}$ $T = 5063$ s $= 84$ minutes And journey will take half of one time period so the time = 42 minutes. OR $a = -1.54 \times 10^{-6} y$, $y = A$ and $\omega^2 = 1.54 \times 10^{-6}$ $\omega = 1.24 \times 10^{-3}$ $Y = A\text{Sin}\omega t$ $A = A\sin\omega t$ so $\omega t = \frac{\pi}{2}$ $t = \frac{\pi}{2\omega}$ $t = \frac{T}{4} = \frac{\pi}{2(1.24 \times 10^{-3})}$ $\frac{T}{4} = 1266$ s $= 21$ minutes $\frac{T}{2} = 1266 \times 2$ $= 2532$ s $= 42$ minutes	Combination of $f = \frac{1}{T}$ and $\omega = 2\pi f$ to get $T = \frac{2\pi}{\omega}$ OR $\omega = \sqrt{1.54 \times 10^{-6}}$ OR Combination of $\omega = \sqrt{\frac{g}{R_{Earth}}}$ and $T = \frac{2\pi}{\omega}$ to get $T = 2\pi\sqrt{\frac{R_{Earth}}{g}}$ OR Incorrect answer but shows half the Time period. OR $Y = A\text{Sin}\omega t$ $A = A\sin\omega t$ so $\omega t = \frac{\pi}{2}$ $t = \frac{\pi}{2\omega}$	Combination of $f = \frac{1}{T}$ and $\omega = 2\pi f$ to get $T = \frac{2\pi}{\omega}$ AND $\omega = \sqrt{1.54 \times 10^{-6}}$ to get 5063 s (84 minutes). OR Combination of $\omega = \sqrt{\frac{g}{R_{Earth}}}$ and $T = \frac{2\pi}{\omega}$ to get $T = 2\pi\sqrt{\frac{R_{Earth}}{g}} = 2\pi\sqrt{\frac{6.38 \times 10^6}{9.81}}$ T = 5063 s = 84 minutes OR $a = -1.54 \times 10^{-6} y$, $y = A$ and $\omega^2 = 1.54 \times 10^{-6}$ $\omega = 1.24 \times 10^{-3}$ $Y = A\text{Sin}\omega t$ $A = A\sin\omega t$ so $\omega t = \frac{\pi}{2}$ $t = \frac{\pi}{2\omega}$ $t = \frac{T}{4} = \frac{\pi}{2(1.24 \times 10^{-3})}$ $\frac{T}{4} = 1266$ s $= 21$ minutes	Combination of $f = \frac{1}{T}$ and $\omega = 2\pi f$ to get $T = \frac{2\pi}{\omega}$ Use of $T = \frac{2\pi}{\omega}$ and $\omega^2 = 1.54 \times 10^{-6}$ to get 5063 s (84 minutes) The journey time is 42 minutes (half the Time period). OR Combination of $\omega = \sqrt{\frac{g}{R_{Earth}}}$ and $T = \frac{2\pi}{\omega}$ to get $T = 2\pi\sqrt{\frac{R_{Earth}}{g}} = 2\pi\sqrt{\frac{6.38 \times 10^6}{9.81}}$ T = 5063 s = 84 minutes The journey time is 42 minutes (half the Time period). OR $a = -1.54 \times 10^{-6} y$, $y = A$ and $\omega^2 = 1.54 \times 10^{-6}$ $\omega = 1.24 \times 10^{-3}$ $Y = A\text{Sin}\omega t$ $A = A\sin\omega t$ so $\omega t = \frac{\pi}{2}$ $t = \frac{\pi}{2\omega}$ $t = \frac{T}{4} = \frac{\pi}{2(1.24 \times 10^{-3})}$ $\frac{T}{4} = 1266$ s $= 21$ minutes $\frac{T}{2} = 1266 \times 2$ $= 2532$ s $= 42$ minutes

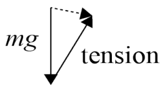
2014(2) (a)	$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1.55}{9.81}} = 2.50\text{ s}$ $t = \frac{1}{2}T = 1.25\text{ s}$	Correct period.		
2013(3) (a)	SHOW THAT QUESTION $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1.2}{9.81}} = 2.1975 = 2.2\text{ s}$	Correct equation OR substitution.		
(b)	Angle of the swing must be small, so that the motion can be approximated as a straight line and the force is proportional to the displacement.	Angle / amplitude must be kept small. The motion can be approximated to a straight line OR force / acceleration are proportional to displacement OR force / acceleration is towards the equilibrium position.	Angle / amplitude / initial displacement is small AND (The motion of the ball can be approximated to a straight line OR force / acceleration is proportional to displacement OR force / acceleration is towards the equilibrium position).	
(c)		Downwards sloping line. Oscillation with constant period that decreases with time.	Line has negative slope with a decreasing size of slope.	

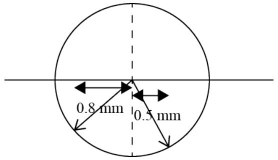
(d)	The ball will start oscillating if the frequency of the shaking matches the natural frequency of the pendulum. If the frequencies are the same, resonance will occur, which means the energy used for the shaking will be transferred into the ball, giving it kinetic energy to make the ball swing.	Shake at (resonant frequency OR natural frequency OR same time each cycle). $E_{Pgrav} \leq \geq E_K$ as ball oscillates. Standing wave set up (or words describing this).	Shaking at (resonant frequency OR natural frequency OR same time each cycle). AND will cause (large amplitude OR large energy transfer OR energy transferred from shake to ball). Wave reflects / interferes and standing wave is set up. Shake at (resonant freq OR natural freq OR same time each cycle) AND ($E_{Pgrav} \geq E_K$ as ball oscillates OR statement of conservation of energy). Force of hand pulls ball (increasing speed / acc / restoring force / amplitude OR causing acceleration / restoring force).	Shaking at (resonant frequency OR natural frequency OR same time each cycle). AND Causes (energy to be transferred from shake to E_{Pgrav} or E_K of ball OR wave reflects and interferes producing standing wave).

(e)	 The forces acting on the ball are the tension force and the gravitational force. The gravitational force is constant, and always acts downwards. The tension force changes direction as the ball swings because it always points in the direction of the cord. At each end of the swing, the tension plus the gravitational force add to make the net force which is the restoring force which is at a tangent to the path. The tension force balances the component of the gravitational force opposite to the direction of the string. The restoring force is the component of the gravitational force that is perpendicular to the string. As the displacement increases, the angle of the string increases, so the size of the restoring force increases.  Note: If students use the word "support" in place of tension, drop mark down by one.	Gravitational force and tension identified as forces acting on the ball. (Symbols sufficient) Gravitational force does not change Note: Accept air resistance included -	Gravitational force and tension named and labelled diagram of tension at correct non-zero angle and gravitational force acting downwards. No incorrect forces (e.g. F_c). Tension increases displacement decreases because the centripetal force is greater because it is moving faster (curved) OR Tension increases as displacement decreases as tension force cancels a component of gravity (curved) OR Tension decreases as displacement decreases as horizontal component of tension gets smaller (straight line) OR Tension decreases as displacement decreases as vertical component cancels gravitational force (straight line) OR Tension changes angle as it always acts towards the cord OR (Restoring force is component of (tension or gravitational force) AND restoring force increases with greater angle) Note: Accept air resistance included.	Labelled diagram showing restoring force created (with triangle OR components OR statement showing restoring force is the addition of gravitational force and tension). No incorrect forces (e.g. F_c). AND Tension increases displacement decreases because the centripetal force is greater because it is moving faster OR Tension increases as displacement decreases as tension force cancels a component of gravity OR Tension decreases as displacement decreases as horizontal component of tension gets smaller OR Tension decreases as displacement decreases as vertical component cancels gravitational force OR Tension changes angle as it always acts towards the cord OR (Restoring force is component of (tension or gravitational force) AND restoring force increases with greater angle) Note: Accept air resistance included.
2012(2) (a)	$a = -\omega^2 y \Rightarrow a = 1.45^2 \times 0.80 = 1.682 = 1.68$ m s^{-2}	[2]Correct answer.		

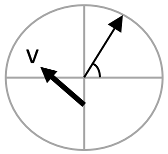
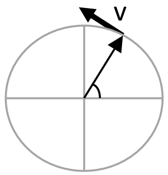
(b)	$y = A \cos \omega t$ $= 0.80 \times \cos(1.45 \times 1.8) = -0.6896$ The negative means Daniel's displacement from the equilibrium position is opposite to when he started, which means he has gone through the equilibrium position. $\Rightarrow$ total distance = A + y = 0.80 + 0.6896 = 1.4896 = 1.49 m	[1]Correct equation selected.	[2]Correct displacement value	[2]Correct distance
(c)	If Daniel had pushed off, his speed as he started his swing would not have been zero and so he would have had kinetic energy. His kinetic energy increase as he drops to the lowest position in his swing would be the same and so his maximum kinetic energy would be greater. This means he would rise further before his kinetic energy has been changed to gravitational potential energy, thus increasing the amplitude of his motion. As a swinging pendulum approximates to simple harmonic motion only if the amplitude is small, increasing the amplitude may mean the motion is no longer simple harmonic.	[1]Explanation shows some idea of: increase in **initial** (angular) speed. OR Some increase in energy. OR Increased amplitude means no longer SHM.	[1]Correct explanation for why the energy has increased. OR Correct explanation for why increase in energy means increase in amplitude. OR Correct explanation for why increased amplitude means no longer simple harmonic motion.	[1]Correct explanation for why the energy has increased and why increase in energy means increase in amplitude. OR Correct explanation for why the energy has increased and correct explanation for why increased amplitude means no longer SHM.
(d)	The period of the pendulum depends on its length ($T = 2\pi\sqrt{(L/g)}$). The length of a pendulum is from the pivot point at the top to the centre of mass of the bob, which is Daniel. If Daniel is sitting down his centre of mass will be further down and so the length of the pendulum will be greater and so the period will be longer.	[1]Idea that the period depends on length and that the length will change.	[1]Correct and full explanation.	

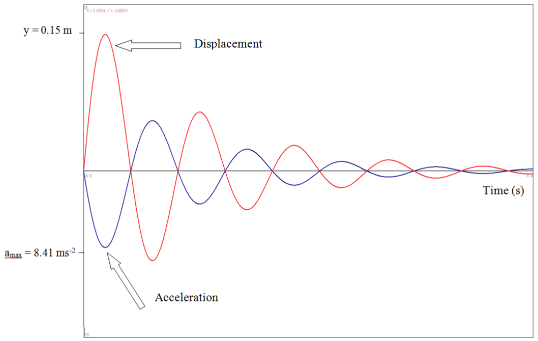
2011(2) (a)	$T = 2\pi\sqrt{\frac{L}{g}}$ $T = 2\pi\sqrt{\frac{5.50}{9.81}}$ $T = 4.7046\ \text{s}$ $T \approx 4.70\ \text{s}$	[2]Correct working. This is a show question.		
(b)(i)	$v_{max} = A\omega$ $\omega = \frac{2\pi}{4.70}$ $\omega = 1.337\ \text{rads}^{-1}$ $v_{max} = A\omega = 0.90 \times 1.337$ $= 1.20\ \text{m s}^{-1}$	[2]Correct ω.	[2]Correct answer.	
(ii)	Equilibrium position shown.	[1]Correct.		

(c)	At all points the tension force has to provide the centripetal force required to keep the bag moving in a circle and balance a component of the force of gravity. At the equilibrium point, the tension is greatest because the speed is greatest and the gravity component is the full gravity force. At the extremes, the speed is zero, so there is no centripetal force, and as one component of the gravity force is providing the restoring force the other component, which must be balanced by the tension force is therefore less than the full gravity force. For both these reasons the tension force at the end positions has decreased. 	[1]Recognition that the tension force provides the centripetal force.	[1]Recognition that tension force must provide both the centripetal force and must balance gravity. OR Explanation for why the centripetal force is greatest at the equilibrium position. **or** why the gravity component that tension must balance is greatest at the equilibrium position.	[1]Explanation for why the centripetal force AND the gravity component that tension must balance are both greatest at the equilibrium position.
(d)	$T = 2\pi\sqrt{\frac{l}{g}} = \frac{2\pi}{\omega}$ $\omega = \frac{v}{r} \propto \sqrt{\frac{1}{l}}$ So increasing length makes v_{max} decrease.	[1]Angular frequency / frequency decreases because the time period is increases.	[1]v decreases because T increases and angular frequency decreases.	

(e)	 $\theta_{tot} = \sin^{-1}\frac{0.8}{0.9} + \sin^{-1}\frac{0.5}{0.9}$ $= 1.68$ rad $t = \frac{\theta_{tot}}{\omega} = 1.68\left(\frac{4.70}{2\pi}\right)$ $= 1.26$ s	[2]Correct diagram.	[2]Correct θ_{tot}. OR Correct time consistent with using r = 5.5.	[2]Correct time.
2010(3) (a)	As $a = -\omega^2 y$, Size of the acceleration must be proportional to the size of the displacement.	[1]Correct description.		
(b)	Friction works against the motion, changing energy to heat. The amplitude depends on the energy of the motion, so as energy is lost, the amplitude decreases.	[1]Recognition that the amplitude decreases	[1]Decrease in amplitude linked to decrease in energy.	[1]Decrease in amplitude linked to decrease in energy which is linked to the action of friction changing energy to heat etc.
(c)	T = $\frac{2\pi}{\omega} = \frac{2\pi}{2.40}$ = 2.61799 $\Rightarrow$n = $\frac{83.8}{2.61799}$ = 32. 01 = 32 oscillations.	[2]Correct T	[2]Correct answer.	
(d)(i)	$F = ma = -m\omega^2 y = 1.65 \times 2.40^2 \times 0.0270$ = 0.2566 = 0.257 N	[2]Correct value for a. OR [1]Correct answer.	[2]Correct answer consistent with using y in centimetres.	[2]Correct working and answer.

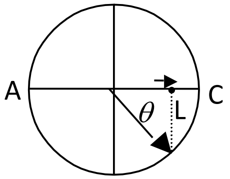
(ii)	v = Aω cos ωt, y = Asinωt $\Rightarrow \omega t = \sin^{-1}\frac{y}{A} = \sin^{-1}\frac{0.027}{0.147} = 0.18472$ $\Rightarrow$ v = 0.147 × 2.62 × cos (0.183673) = 0.34680 = **0.35 m s^{-1}**	[2]Correct ωt or t. OR [1]Correct answer.	[2]Correct answer consistent with using y in centimetres.	[2]Correct working and answer.
2009(3) (a)	THIS IS A SHOW QUESTION k = F/x F = mg = 635 × 9.81(F= 6229 N) k = 635 × 9.81 / (0.175) = 35600 N m^{-1}	[2]Extension = 17.5cm and Force = 6229N	[2]Correct working. k = 35596.286 N m^{-1} OR Finds F or x and substitution OK x in m	
(b)	THIS IS A SHOW QUESTION $T = 2\pi \times \sqrt{\frac{m}{k}}$ $= 2\pi \times \sqrt{\frac{635}{35\,600}} = 0.839$ s	[2]Correct working. T = 0.8392s = 0.8391979s		

(c)	 	[1]Correct orientation of phasor. OR Any description of the planes velocity at this position. This could be a calculation or a description. v(shm) = 0.56 ms^{-1}		
(d)	$a_{max} = A\omega^2$ $\omega = 2\pi/T = 2\pi/0.839 = 7.49$ rad s^{-1} $a_{max} = 0.15 \times 7.492 = 8.41$ m s^{-2} Vertical component at the time shown $a = a_{max} \times \cos 30° = -7.28$ m s^{-2}.		[2]Correct $a_{max} = 8.41$ OR Correct y value $y = 0.15 \sin 60^0$ $= 0.13$ m	[2]Correct answer $a = 7.28$ m s^{-2}.

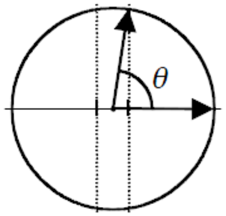
(e)	The shock absorbers reduce the amplitude of the SHM so the displacement-time and acceleration-time graphs would show a gradual decrease in amplitude (period and ω is (roughly) constant) so Since $a = -\omega^2 y$ $a \propto -y$ The discomfort of passengers is due to acceleration (constant velocity would not be noticeable).  	Describe the effect of moderate damping on: • displacement • acceleration how it affects discomfort [1]reduced displacement OR Graph of d vs t vertical axis labelled or title **OR** reduced acceleration OR Graph of a vs t	Discomfort is proportional to acceleration or net force **OR** [1]reduced displacement OR Graph of d vs t vertical axis labelled or title **AND** reduced acceleration OR Graph of a vs t	[1]Discomfort is proportional to acceleration or net force **AND** [1]reduced displacement OR Graph of d vs t (approx same T) **AND** why acceleration is reduced: a α (–) y
2008(3) (a)	$T = 2\pi\sqrt{\frac{l}{g}}$ $T = 2\pi\sqrt{\frac{2.1}{9.81}} = 2\pi \times 0.46291 = 2.9070 = 2.9 \text{ s}$	A **SHOW** question [2]Correct substitution into correct equation		

2007(3) (a)	$T = \frac{9.32}{13}$ = 0.716923 = **0.717 s**	[2]Correct answer [1]Answer given to 3 sf and FOUR answers given with correct units.		
(b)	$T = 2\pi\sqrt{\frac{R}{g}} \Rightarrow R = \frac{0.716923^2 \times 9.81}{(2\pi)^2}$ = 0.127171 = **0.13 m**	[2]Correct answer consequential to answer given in 3(a).		
(c)	The restoring force is proportional to the displacement / in the opposite direction to the displacement / acts towards the equilibrium position.	[1]One statement is given.		
(d)	$F_R = \frac{F_g y}{R}$, and $F_R = ma$ but $a = \omega^2 y = \left(\frac{2\pi}{T}\right)^2$ and $F_g = mg$ $\Rightarrow \frac{mgy}{R} = my\left(\frac{2\pi}{T}\right)^2$ $\Rightarrow \quad T = 2\pi\sqrt{\frac{R}{g}}$	[1]Correct answer shows recognition that F_R also equals ma.		[2]Correct working
(e)	Friction with the air and against the surface.	[1]Must state friction		
(f)	Because of the changed shape of the surface the forces opposing the bowl's motion will be greater. This means that the rate at which the SHM energy is dissipated will be greater.	[1]Idea of increased forces acting against the bowl / idea of greater rate of change of energy.	[1]Idea of increased forces creating more energy change / of how a soft surface causes a greater rate of change of energy.	[1]Increased opposing forces clearly linked to greater rate of change of energy.

2006(1) (a)	$\omega = 2\pi f$ $\Rightarrow f = \frac{2.2}{2\pi}$ = 0.35014 = 0.35 s^{-1} (Hz)	[2]Correct answer. Has to be a calculation with ω.		
(b)	$T = T = 2\pi\sqrt{\frac{l}{g}}$ $2.9 = 2 \times \pi \times \sqrt{\left(\frac{l}{9.81}\right)}$ $\Rightarrow l$ = 2.08980 = 2.1 m	[2]Correct answer.		
(c)	As the length is shortened, *T* will also decrease. If *L* is halved, *T* will decrease by square root of 2. T = 2.056 s	[1]Idea that *T* will decrease if *L* decreases.	[1]Idea that *T* will decrease if *L* decreases, and the appropriate factor is given. Calculation of T can be included with the explanation.	
(d)	$a_{max} = +/-A\omega^2$ $0.37 \times 2.2^2 = 1.7908$ $a_{max} = 1.8 \text{ m s}^{-2}$	[2]Correct answer.		
(e)	Arrow drawn at the tangent to the path in the direction of the equilibrium position.	[1]Arrow drawn in correct direction.	[1]Arrow drawn on tangent in correct direction.	

(f)	$F = m\omega^2 y = 31 \times 2.2^2 \times 0.25$ $= 37.51 = 38$ N		[2]Correct answer. Accept if method used is to find the component of the gravity force.	
(g)		$\theta = \omega t$ $\cos\omega t = \frac{.37 - .12}{.37}$ $\Rightarrow \omega t = 0.828915$ (47.5^0) $\Rightarrow t = 0.377$ s $t_{total} = t + ½T$ = 1.826 = 1.8 s	[1]Ability to relate the angular displacement of a phasor to a linear displacement of a SHM particle.	[2]Calculation of t = 0.377 s
2005(2) (a)	$A = \frac{1}{2} \times 0.150 = 0.0750$ **0.0750 m**	Correct answer		
(b)	$a = \omega^2 A = 1.8^2 \times 0.0750$ $= 0.243$ **0.24 m s^{-2}**	Rounded to 2 sig. fig. plus 6 answers given with correct unit.	Correct answer.	
(c)	0.075 m / at the amplitude / at maximum displacement.	Correct statement.		

(d)	As the timing starts from the end position the "sine" form of the formula must be used. $= 1.5 \times 1.8 \times \sin(1.8 \times 0.75)$ $= 2.63445$ **2.6 m s^{-1}**	Recognition that the "sine" form of the formula must be used.	Correct answer consistent with calculator in degree mode.	Correct answer
(e)	$a = \omega^2 y$. This shows that the acceleration depends only on the displacement and angular speed. As the natural frequency is the same for both damped and undamped motion, the angular speed will stay the same.	One correct and relevant statement: a is proportional to y / ω is constant.	Recognition that acceleration depends on displacement **only**.	Link made between the acceleration depending on displacement and angular speed only, and constant angular speed implies ω depends on y only.
(f)	$T = 2\pi/\omega \quad = 2\pi/1.8 \quad = 3.49066$ **3.5 s**		Correct answer	

(g)	 Every $\frac{1}{4}$ cycle the displacement phasor rotates through angle θ while the displacement is outside the maximum damped displacement. $\cos\theta = \frac{\frac{1}{2}\times 0.150}{\frac{1}{2}\times 3.0}$ $= 87.134°$ (1.52078 rad) total angle turned by displacement phasor in a complete cycle = $4 \times \theta$ = 348.536° (6.08310) $t = \frac{4\theta}{360} \times T$ $(t = \frac{4\theta}{\omega})$ $t = 3.37950$ $(t = 3.3795)$ = **3.4 s**		Correctly calculated θ.	Correct answer.